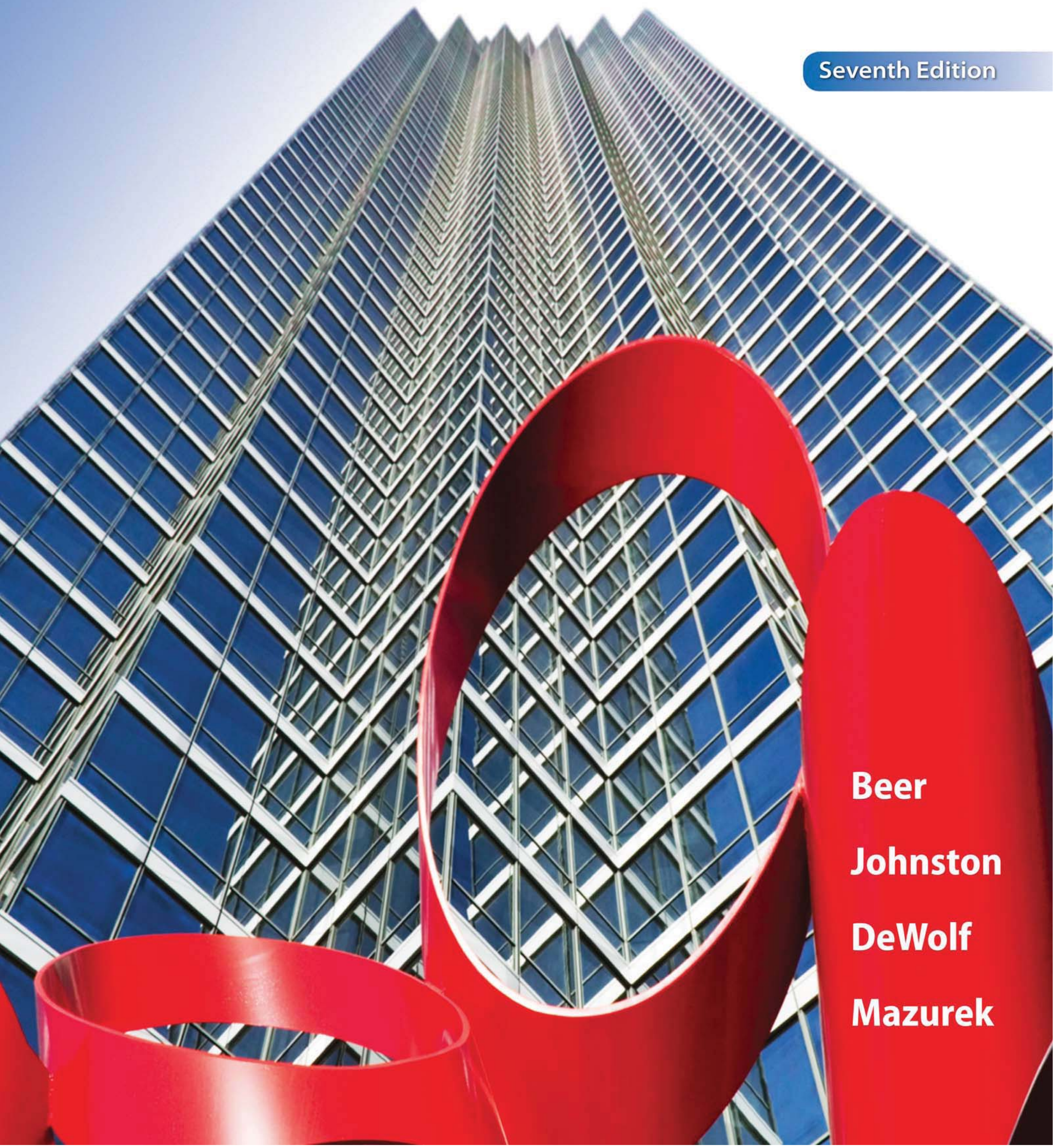
MECHANICS of MATERIALS

Seventh Edition



Beer
Johnston
DeWolf
Mazurek

Seventh Edition

Mechanics of Materials

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MECHANICS OF MATERIALS, SEVENTH EDITION

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The photo on the cover shows the steel sculpture "Venture" by Alex Liberman (1912-1999) in front of the Bank of America Building in Dallas, Texas. The building is supported by a combination of structural steel and reinforced concrete.

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About the Authors

John T. DeWolf, Professor of Civil Engineering at the University of Connecticut, joined the Beer and Johnston team as an author on the second edition of *Mechanics of Materials*. John holds a B.S. degree in civil engineering from the University of Hawaii and M.E. and Ph.D. degrees in structural engineering from Cornell University. He is a Fellow of the American Society of Civil Engineers and a member of the Connecticut Academy of Science and Engineering. He is a registered Professional Engineer and a member of the Connecticut Board of Professional Engineers. He was selected as a University of Connecticut Teaching Fellow in 2006. Professional interests include elastic stability, bridge monitoring, and structural analysis and design.

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Preface

Objectives

The main objective of a basic mechanics course should be to develop in the engineering student the ability to analyze a given problem in a simple and logical manner and to apply to its solution a few fundamental and well-understood principles. This text is designed for the first course in mechanics of materials—or strength of materials—offered to engineering students in the sophomore or junior year. The authors hope that it will help instructors achieve this goal in that particular course in the same way that their other texts may have helped them in statics and dynamics. To assist in this goal, the seventh edition has undergone a complete edit of the language to make the book easier to read.

General Approach

In this text the study of the mechanics of materials is based on the understanding of a few basic concepts and on the use of simplified models. This approach makes it possible to develop all the necessary formulas in a rational and logical manner, and to indicate clearly the conditions under which they can be safely applied to the analysis and design of actual engineering structures and machine components.

Free-body Diagrams Are Used Extensively. Throughout the text free-body diagrams are used to determine external or internal forces. The use of "picture equations" will also help the students understand the superposition of loadings and the resulting stresses and deformations.

NEW **The SMART Problem-Solving Methodology is Employed.** New to this edition of the text, students are introduced to the SMART approach for solving engineering problems, whose acronym reflects the solution steps of ***S***trategy, ***M***odeling, ***A***nalysis, and ***R***eflect & ***T***hink. This methodology is used in all Sample Problems, and it is intended that students will apply this approach in the solution of all assigned problems.

Design Concepts Are Discussed Throughout the Text Whenever Appropriate. A discussion of the application of the factor of safety to design can be found in Chap. 1, where the concepts of both allowable stress design and load and resistance factor design are presented.

A Careful Balance Between SI and U.S. Customary Units Is Consistently Maintained. Because it is essential that students be able to handle effectively both SI metric units and U.S. customary units, half the concept applications, sample problems, and problems to be assigned have been stated in SI units and half in U.S. customary units. Since a large number of problems are available, instructors can assign problems using each system of units in whatever proportion they find desirable for their class.

Optional Sections Offer Advanced or Specialty Topics. Topics such as residual stresses, torsion of noncircular and thin-walled members, bending of curved beams, shearing stresses in non-symmetrical members, and failure criteria have been included in optional sections for use in courses of varying emphases. To preserve the integrity of the subject, these topics are presented in the proper sequence, wherever they logically belong. Thus, even when not covered in the course, these sections are highly visible and can be easily referred to by the students if needed in a later course or in engineering practice. For convenience all optional sections have been indicated by asterisks.

Chapter Organization

It is expected that students using this text will have completed a course in statics. However, Chap. 1 is designed to provide them with an opportunity to review the concepts learned in that course, while shear and bending-moment diagrams are covered in detail in Secs. 5.1 and 5.2. The properties of moments and centroids of areas are described in Appendix A; this material can be used to reinforce the discussion of the determination of normal and shearing stresses in beams (Chaps. 4, 5, and 6).

The first four chapters of the text are devoted to the analysis of the stresses and of the corresponding deformations in various structural members, considering successively axial loading, torsion, and pure bending. Each analysis is based on a few basic concepts: namely, the conditions of equilibrium of the forces exerted on the member, the relations existing between stress and strain in the material, and the conditions imposed by the supports and loading of the member. The study of each type of loading is complemented by a large number of concept applications, sample problems, and problems to be assigned, all designed to strengthen the students' understanding of the subject.

The concept of stress at a point is introduced in Chap. 1, where it is shown that an axial load can produce shearing stresses as well as normal stresses, depending upon the section considered. The fact that stresses depend upon the orientation of the surface on which they are computed is emphasized again in Chaps. 3 and 4 in the cases of torsion and pure bending. However, the discussion of computational techniques—such as Mohr's circle—used for the transformation of stress at a point is delayed until Chap. 7, after students have had the opportunity to solve problems involving a combination of the basic loadings and have discovered for themselves the need for such techniques.

The discussion in Chap. 2 of the relation between stress and strain in various materials includes fiber-reinforced composite materials. Also, the study of beams under transverse loads is covered in two separate chapters. Chapter 5 is devoted to the determination of the normal stresses in a beam and to the design of beams based on the allowable normal stress in the material used (Sec. 5.3). The chapter begins with a discussion of the shear and bending-moment diagrams (Secs. 5.1 and 5.2) and includes an optional section on the use of singularity functions for the determination of the shear and bending moment in a beam (Sec. 5.4). The chapter ends with an optional section on nonprismatic beams (Sec. 5.5).

Chapter 6 is devoted to the determination of shearing stresses in beams and thin-walled members under transverse loadings. The formula for the shear flow, $q = VQ/I$, is derived in the traditional way. More advanced aspects of the design of beams, such as the determination of the principal stresses at the junction of the flange and web of a W-beam, are considered in Chap. 8, an optional chapter that may be covered after the transformations of stresses have been discussed in Chap. 7. The design of transmission shafts is in that chapter for the same reason, as well as the determination of stresses under combined loadings that can now include the determination of the principal stresses, principal planes, and maximum shearing stress at a given point.

Statically indeterminate problems are first discussed in Chap. 2 and considered throughout the text for the various loading conditions encountered. Thus, students are presented at an early stage with a method of solution that combines the analysis of deformations with the conventional analysis of forces used in statics. In this way, they will have become thoroughly familiar with this fundamental method by the end of the course. In addition, this approach helps the students realize that stresses themselves are statically indeterminate and can be computed only by considering the corresponding distribution of strains.

The concept of plastic deformation is introduced in Chap. 2, where it is applied to the analysis of members under axial loading. Problems involving the plastic deformation of circular shafts and of prismatic beams are also considered in optional sections of Chaps. 3, 4, and 6. While some of this material can be omitted at the choice of the instructor, its inclusion in the body of the text will help students realize the limitations of the assumption of a linear stress-strain relation and serve to caution them against the inappropriate use of the elastic torsion and flexure formulas.

The determination of the deflection of beams is discussed in Chap. 9. The first part of the chapter is devoted to the integration method and to the method of superposition, with an optional section (Sec. 9.3) based on the use of singularity functions. (This section should be used only if Sec. 5.4 was covered earlier.) The second part of Chap. 9 is optional. It presents the moment-area method in two lessons.

Chapter 10, which is devoted to columns, contains material on the design of steel, aluminum, and wood columns. Chapter 11 covers energy methods, including Castigliano's theorem.

Supplemental Resources for Instructors

Find the **Companion Website** for Mechanics of Materials at www.mhhe.com/beerjohnston. Included on the website are lecture PowerPoints, an image library, and animations. On the site you'll also find the **Instructor's Solutions Manual** (password-protected and available to instructors only) that accompanies the seventh edition. The manual continues the tradition of exceptional accuracy and normally keeps solutions contained to a single page for easier reference. The manual includes an in-depth review of the material in each chapter and houses tables designed to assist instructors in creating a schedule of assignments for their courses. The various topics covered in the text are listed in Table I, and a suggested number of periods to be spent on each topic is indicated. Table II provides a brief description of all groups of problems and a classification of the problems in each group according to the units used. A Course Organization Guide providing sample assignment schedules is also found on the website.

Via the website, instructors can also request access to **C.O.S.M.O.S.**, the Complete Online Solutions Manual Organization System that allows instructors to create custom homework, quizzes, and tests using end-of-chapter problems from the text.

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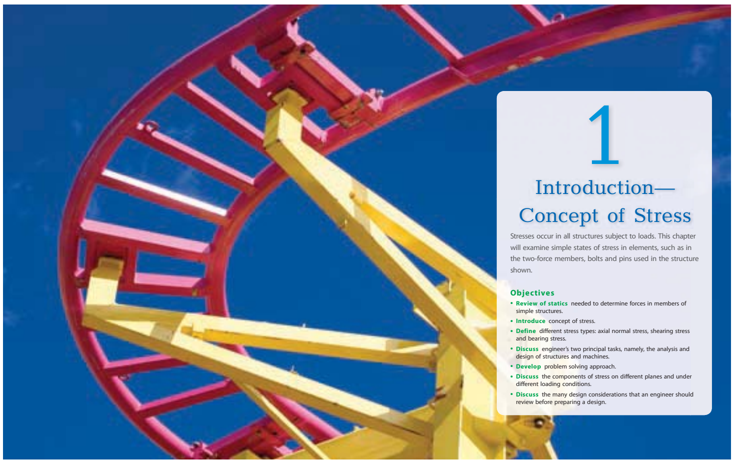
We also gratefully acknowledge the help, comments, and suggestions offered by the many reviewers and users of previous editions of *Mechanics of Materials*.

John T. DeWolf
David F. Mazurek

Guided Tour

Chapter Introduction. Each chapter begins with an introductory section that sets up the purpose and goals of the chapter, describing in simple terms the material that will be covered and its application to the solution of engineering problems. Chapter Objectives provide students with a preview of chapter topics.



Chapter Lessons. The body of the text is divided into units, each consisting of one or several theory sections, Concept Applications, one or several Sample Problems, and a large number of homework problems. The Companion Website contains a Course Organization Guide with suggestions on each chapter lesson.

Concept Applications. Concept Applications are used extensively within individual theory sections to focus on specific topics, and they are designed to illustrate specific material being presented and facilitate its understanding.

Concept Application 1.1

Considering the structure of Fig. 1.1 on page 5, assume that rod BC is made of a steel with a maximum allowable stress $\sigma_{all} = 165$ MPa. Can rod BC safely support the load to which it will be subjected? The magnitude of the force F_{BC} in the rod was 50 kN. Recalling that the diameter of the rod is 20 mm, use Eq. (1.5) to determine the stress created in the rod by the given loading.

$$P = F_{BC} = +50 \text{ kN} = +50 \times 10^3 \text{ N}$$

$$A = \pi r^2 = \pi\left(\frac{20 \text{ mm}}{2}\right)^2 = \pi(10 \times 10^{-3} \text{ m})^2 = 314 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{+50 \times 10^3 \text{ N}}{314 \times 10^{-6} \text{ m}^2} = +159 \times 10^6 \text{ Pa} = +159 \text{ MPa}$$

Since σ is smaller than σ_{all} of the allowable stress in the steel used, rod BC can safely support the load.

Sample Problems. The Sample Problems are intended to show more comprehensive applications of the theory to the solution of engineering problems, and they employ the SMART problem-solving methodology that students are encouraged to use in the solution of their assigned problems. Since the sample problems have been set up in much the same form that students will use in solving the assigned problems, they serve the double purpose of amplifying the text and demonstrating the type of neat and orderly work that students should cultivate in their own solutions. In addition, in-problem references and captions have been added to the sample problem figures for contextual linkage to the step-by-step solution.

Sample Problem 1.2

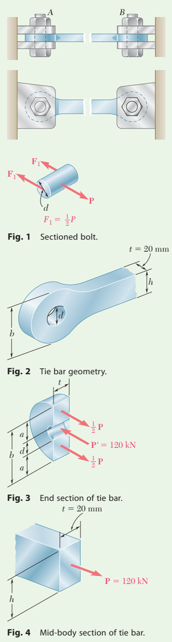


Fig. 1 Sectioned bolt.

Fig. 2 Tie bar geometry.

Fig. 3 End section of tie bar.

Fig. 4 Mid-body section of tie bar.

The steel tie bar shown is to be designed to carry a tension force of magnitude $P = 120$ kN when bolted between double brackets at A and B. The bar will be fabricated from 20-mm-thick plate stock. For the grade of steel to be used, the maximum allowable stresses are $\sigma = 175$ MPa, $\tau = 100$ MPa, and $\sigma_b = 350$ MPa. Design the tie bar by determining the required values of (a) the diameter d of the bolt, (b) the dimension b at each end of the bar, and (c) the dimension h of the bar.

STRATEGY: Use free-body diagrams to determine the forces needed to obtain the stresses in terms of the design tension force. Setting these stresses equal to the allowable stresses provides for the determination of the required dimensions.

MODELING and ANALYSIS:

a. Diameter of the Bolt. Since the bolt is in double shear (Fig. 1), $F_1 = \frac{1}{2}P = 60$ kN.

$$\tau = \frac{F_1}{A} = \frac{60 \text{ kN}}{\frac{1}{4}\pi d^2} \qquad 100 \text{ MPa} = \frac{60 \text{ kN}}{\frac{1}{4}\pi d^2} \qquad d = 27.6 \text{ mm}$$

Use $d = 28$ mm ◀

At this point, check the bearing stress between the 20-mm-thick plate (Fig. 2) and the 28-mm-diameter bolt.

$$\sigma_b = \frac{P}{td} = \frac{120 \text{ kN}}{(0.020 \text{ m})(0.028 \text{ m})} = 214 \text{ MPa} < 350 \text{ MPa} \qquad \text{OK}$$

b. Dimension *b* at Each End of the Bar. We consider one of the end portions of the bar in Fig. 3. Recalling that the thickness of the steel plate is $t = 20$ mm and that the average tensile stress must not exceed 175 MPa, write

$$\sigma = \frac{\frac{1}{2}P}{ta} \qquad 175 \text{ MPa} = \frac{60 \text{ kN}}{(0.02 \text{ m})a} \qquad a = 17.14 \text{ mm}$$

$$b = d + 2a = 28 \text{ mm} + 2(17.14 \text{ mm}) \qquad b = 62.3 \text{ mm} ◀$$

c. Dimension *h* of the Bar. We consider a section in the central portion of the bar (Fig. 4). Recalling that the thickness of the steel plate is $t = 20$ mm, we have

$$\sigma = \frac{P}{th} \qquad 175 \text{ MPa} = \frac{120 \text{ kN}}{(0.020 \text{ m})h} \qquad h = 34.3 \text{ mm}$$

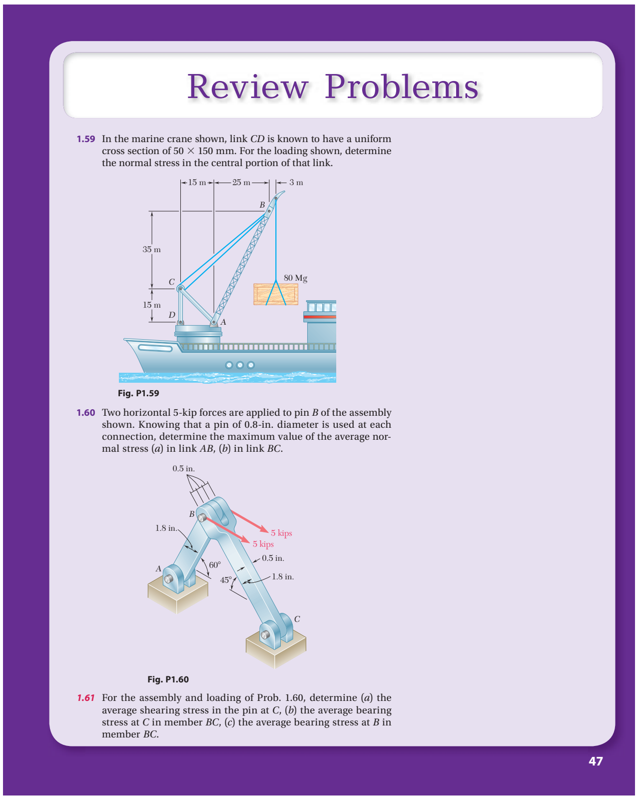
Use $h = 35$ mm ◀

REFLECT and THINK: We sized d based on bolt shear, and then checked bearing on the tie bar. Had the maximum allowable bearing stress been exceeded, we would have had to recalculate d based on the bearing criterion.

Homework Problem Sets. Over 25% of the nearly 1500 homework problems are new or updated. Most of the problems are of a practical nature and should appeal to engineering students. They are primarily designed, however, to illustrate the material presented in the text and to help students understand the principles used in mechanics of materials. The problems are grouped according to the portions of material they illustrate and are arranged in order of increasing difficulty. Answers to a majority of the problems are given at the end of the book. Problems for which the answers are given are set in blue type in the text, while problems for which no answer is given are set in red.

Chapter Review and Summary. Each chapter ends with a review and summary of the material covered in that chapter. Subtitles are used to help students organize their review work, and cross-references have been included to help them find the portions of material requiring their special attention.

Review Problems. A set of review problems is included at the end of each chapter. These problems provide students further opportunity to apply the most important concepts introduced in the chapter.



Review Problems

1.59 In the marine crane shown, link *CD* is known to have a uniform cross section of 50 × 150 mm. For the loading shown, determine the normal stress in the central portion of that link.

Fig. P1.59

1.60 Two horizontal 5-kip forces are applied to pin *B* of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link *AB*, (*b*) in link *BC*.

Fig. P1.60

1.61 For the assembly and loading of Prob. 1.60, determine (*a*) the average shearing stress in the pin at *C*, (*b*) the average bearing stress at *C* in member *BC*, (*c*) the average bearing stress at *B* in member *BC*.

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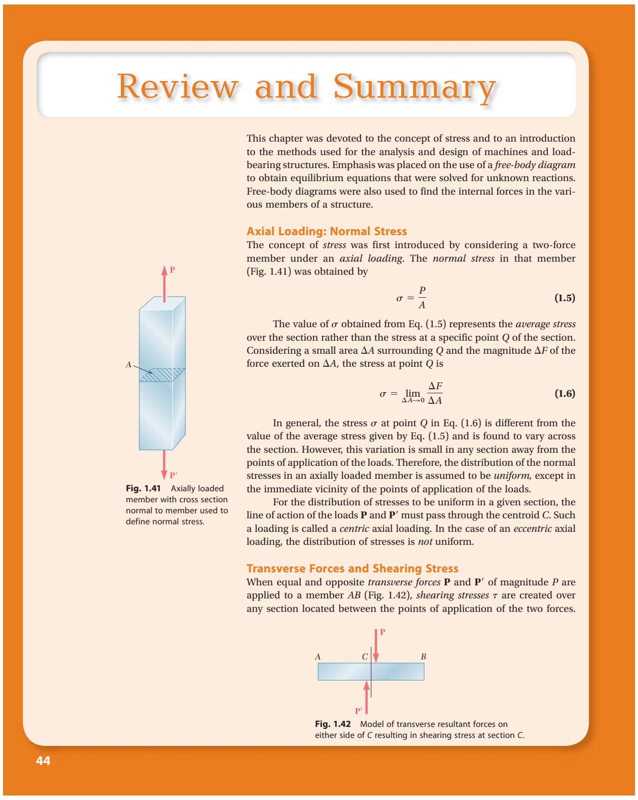
Computer Problems. Computers make it possible for engineering students to solve a great number of challenging problems. A group of six or more problems designed to be solved with a computer can be found at the end of each chapter. These problems can be solved using any computer language that provides a basis for analytical calculations. Developing the algorithm required to solve a given problem will benefit the students in two different ways: (1) it will help them gain a better understanding of the mechanics principles involved; (2) it will provide them with an opportunity to apply the skills acquired in their computer programming course to the solution of a meaningful engineering problem.



Review and Summary

This chapter was devoted to the concept of stress and to an introduction to the methods used for the analysis and design of machines and load-bearing structures. Emphasis was placed on the use of a *free-body diagram* to obtain equilibrium equations that were solved for unknown reactions. Free-body diagrams were also used to find the internal forces in the various members of a structure.

Axial Loading: Normal Stress

The concept of *stress* was first introduced by considering a two-force member under an *axial loading*. The *normal stress* in that member (Fig. 1.41) was obtained by

$$\sigma = \frac{P}{A} \quad \textbf{(1.5)}$$

The value of σ obtained from Eq. (1.5) represents the *average stress* over the section rather than the stress at a specific point Q of the section. Considering a small area ΔA surrounding Q and the magnitude ΔF of the force exerted on ΔA, the stress at point Q is

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \quad \textbf{(1.6)}$$

In general, the stress σ at point Q in Eq. (1.6) is different from the value of the average stress given by Eq. (1.5) and is found to vary across the section. However, this variation is small in any section away from the points of application of the loads. Therefore, the distribution of the normal stresses in an axially loaded member is assumed to be *uniform,* except in the immediate vicinity of the points of application of the loads.

For the distribution of stresses to be uniform in a given section, the line of action of the loads **P** and **P′** must pass through the centroid *C*. Such a loading is called a *centric* axial loading. In the case of an *eccentric* axial loading, the distribution of stresses is *not* uniform.

Fig. 1.41 Axially loaded member with cross section normal to member used to define normal stress.

Transverse Forces and Shearing Stress

When equal and opposite *transverse forces* **P** and **P′** of magnitude P are applied to a member *AB* (Fig. 1.42), *shearing stresses* τ are created over any section located between the points of application of the two forces.

Fig. 1.42 Model of transverse resultant forces on either side of *C* resulting in shearing stress at section *C*.

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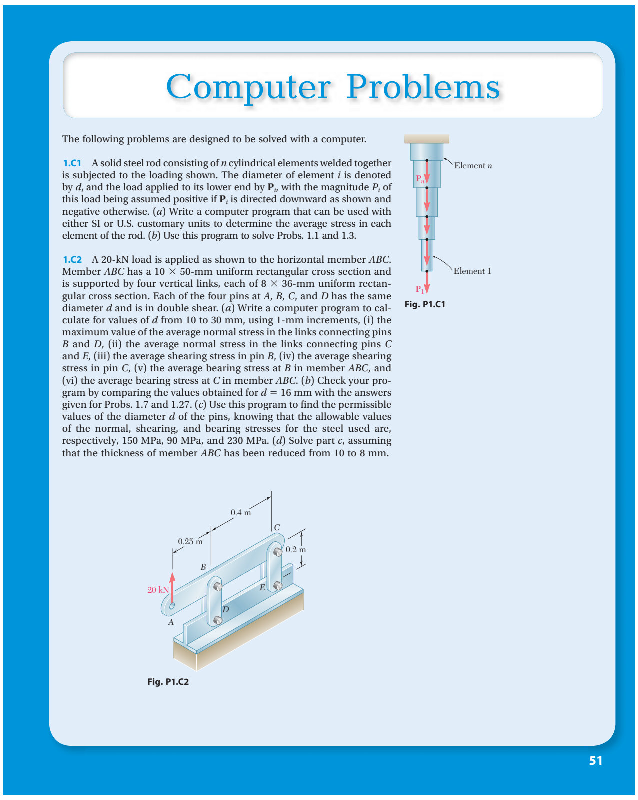

Computer Problems

The following problems are designed to be solved with a computer.

1.C1 A solid steel rod consisting of n cylindrical elements welded together is subjected to the loading shown. The diameter of element i is denoted by d_i and the load applied to its lower end by $\mathbf{P}_i$, with the magnitude P_i of this load being assumed positive if $\mathbf{P}_i$ is directed downward as shown and negative otherwise. (*a*) Write a computer program that can be used with either SI or U.S. customary units to determine the average stress in each element of the rod. (*b*) Use this program to solve Probs. 1.1 and 1.3.

Fig. P1.C1

1.C2 A 20-kN load is applied as shown to the horizontal member *ABC*. Member *ABC* has a 10 × 50-mm uniform rectangular cross section and is supported by four vertical links, each of 8 × 36-mm uniform rectangular cross section. Each of the four pins at *A*, *B*, *C*, and *D* has the same diameter *d* and is in double shear. (*a*) Write a computer program to calculate for values of *d* from 10 to 30 mm, using 1-mm increments, (i) the maximum value of the average normal stress in the links connecting pins *B* and *D*, (ii) the average normal stress in the links connecting pins *C* and *E*, (iii) the average shearing stress in pin *B*, (iv) the average shearing stress in pin *C*, (v) the average bearing stress at *B* in member *ABC*, and (vi) the average bearing stress at *C* in member *ABC*. (*b*) Check your program by comparing the values obtained for $d = 16$ mm with the answers given for Probs. 1.7 and 1.27. (*c*) Use this program to find the permissible values of the diameter *d* of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively, 150 MPa, 90 MPa, and 230 MPa. (*d*) Solve part *c*, assuming that the thickness of member *ABC* has been reduced from 10 to 8 mm.

Fig. P1.C2

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List of Symbols

a	Constant; distance
$\mathbf{A, B, C}, \ldots$	Forces; reactions
$A, B, C, \ldots$	Points
$A, \mathcal{A}$	Area
b	Distance; width
c	Constant; distance; radius
C	Centroid
$C_1, C_2, \ldots$	Constants of integration
C_P	Column stability factor
d	Distance; diameter; depth
D	Diameter
e	Distance; eccentricity; dilatation
E	Modulus of elasticity
f	Frequency; function
$\mathbf{F}$	Force
$F.S.$	Factor of safety
G	Modulus of rigidity; shear modulus
h	Distance; height
$\mathbf{H}$	Force
H, J, K	Points
$I, I_x, \ldots$	Moment of inertia
$I_{xy}, \ldots$	Product of inertia
J	Polar moment of inertia
k	Spring constant; shape factor; bulk modulus; constant
K	Stress concentration factor; torsional spring constant
l	Length; span
L	Length; span
L_e	Effective length
m	Mass
$\mathbf{M}$	Couple
$M, M_x, \ldots$	Bending moment
M_D	Bending moment, dead load (LRFD)
M_L	Bending moment, live load (LRFD)
M_U	Bending moment, ultimate load (LRFD)
n	Number; ratio of moduli of elasticity; normal direction
p	Pressure
$\mathbf{P}$	Force; concentrated load
P_D	Dead load (LRFD)
P_L	Live load (LRFD)
P_U	Ultimate load (LRFD)
q	Shearing force per unit length; shear flow
$\mathbf{Q}$	Force
Q	First moment of area
r	Radius; radius of gyration
$\mathbf{R}$	Force; reaction
R	Radius; modulus of rupture
s	Length
S	Elastic section modulus
t	Thickness; distance; tangential deviation
$\mathbf{T}$	Torque
T	Temperature
u, v	Rectangular coordinates
u	Strain-energy density
U	Strain energy; work
$\mathbf{v}$	Velocity
$\mathbf{V}$	Shearing force
V	Volume; shear
w	Width; distance; load per unit length
$\mathbf{W}, W$	Weight, load
x, y, z	Rectangular coordinates; distance; displacements; deflections
$\bar{x}, \bar{y}, \bar{z}$	Coordinates of centroid
Z	Plastic section modulus
α, β, γ	Angles
α	Coefficient of thermal expansion; influence coefficient
γ	Shearing strain; specific weight
γ_D	Load factor, dead load (LRFD)
γ_L	Load factor, live load (LRFD)
δ	Deformation; displacement
ϵ	Normal strain
θ	Angle; slope
λ	Direction cosine
ν	Poisson's ratio
ρ	Radius of curvature; distance; density
σ	Normal stress
τ	Shearing stress
ϕ	Angle; angle of twist; resistance factor
ω	Angular velocity

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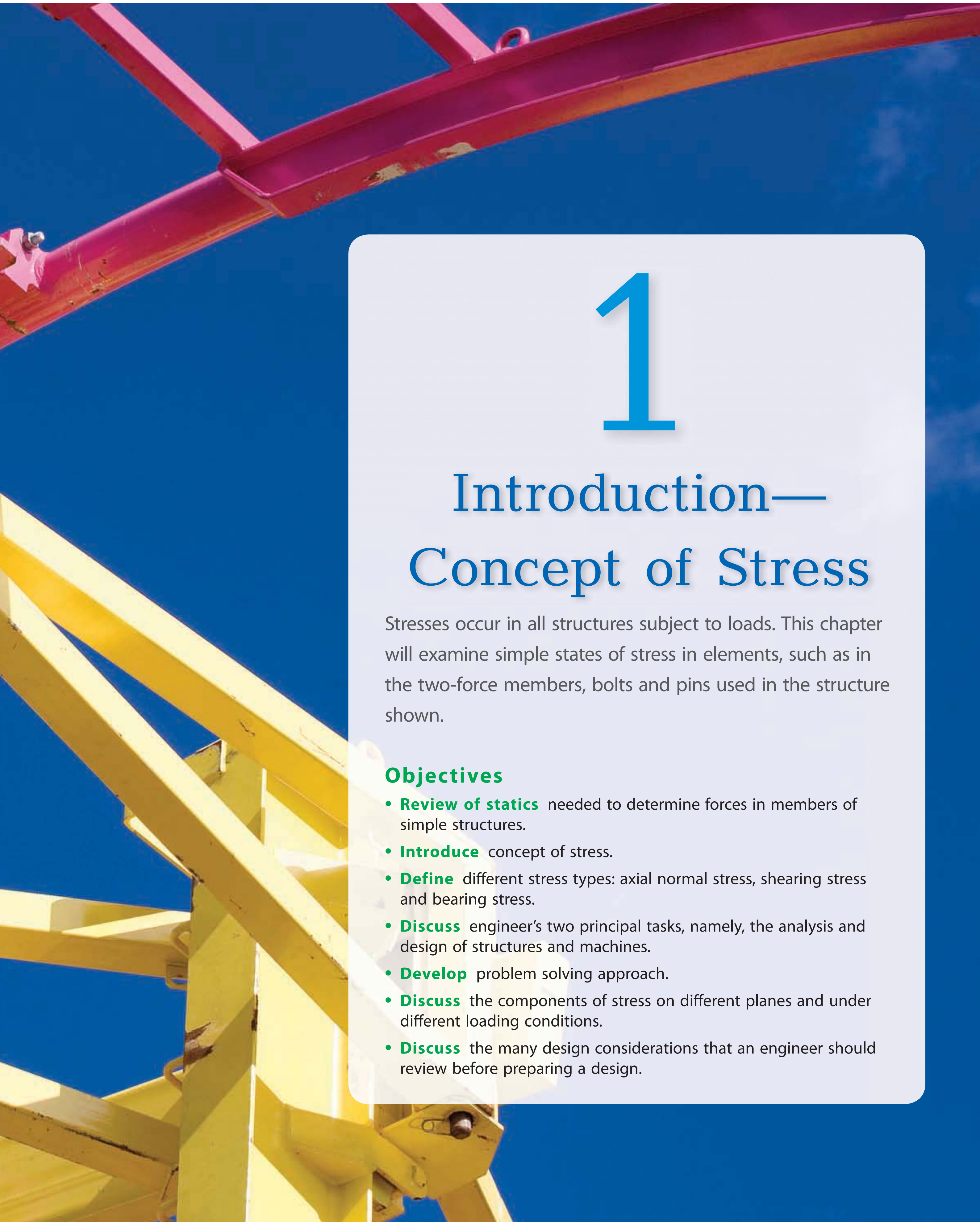
Seventh Edition

Mechanics of Materials



1

Introduction—Concept of Stress

Stresses occur in all structures subject to loads. This chapter will examine simple states of stress in elements, such as in the two-force members, bolts and pins used in the structure shown.

Objectives

- **Review of statics** needed to determine forces in members of simple structures.
- **Introduce** concept of stress.
- **Define** different stress types: axial normal stress, shearing stress and bearing stress.
- **Discuss** engineer's two principal tasks, namely, the analysis and design of structures and machines.
- **Develop** problem solving approach.
- **Discuss** the components of stress on different planes and under different loading conditions.
- **Discuss** the many design considerations that an engineer should review before preparing a design.

Introduction

Photo 1.1 Crane booms used to load and unload ships.

Introduction

The study of mechanics of materials provides future engineers with the means of analyzing and designing various machines and load-bearing structures involving the determination of *stresses* and *deformations*. This first chapter is devoted to the concept of *stress*.

Section 1.1 is a short review of the basic methods of statics and their application to determine the forces in the members of a simple structure consisting of pin-connected members. The concept of *stress* in a member of a structure and how that stress can be determined from the *force* in the member will be discussed in Sec. 1.2. You will consider the *normal stresses* in a member under axial loading, the *shearing stresses* caused by the application of equal and opposite transverse forces, and the *bearing stresses* created by bolts and pins in the members they connect.

Section 1.2 ends with a description of the method you should use in the solution of an assigned problem and a discussion of the numerical accuracy. These concepts will be applied in the analysis of the members of the simple structure considered earlier.

Again, a two-force member under axial loading is observed in Sec. 1.3 where the stresses on an *oblique* plane include both *normal* and *shearing* stresses, while Sec. 1.4 discusses that *six components* are required to describe the state of stress at a point in a body under the most general loading conditions.

Finally, Sec. 1.5 is devoted to the determination of the *ultimate strength* from test specimens and the use of a *factor of safety* to compute the *allowable load* for a structural component made of that material.

1.1 REVIEW OF THE METHODS OF STATICS

Consider the structure shown in Fig. 1.1, which was designed to support a 30-kN load. It consists of a boom *AB* with a 30 × 50-mm rectangular cross section and a rod *BC* with a 20-mm-diameter circular cross section. These are connected by a pin at *B* and are supported by pins and brackets at *A* and *C*, respectively. First draw a *free-body diagram* of the structure by detaching it from its supports at *A* and *C* and showing the reactions that these supports exert on the structure (Fig. 1.2). Note that the sketch of the structure has been simplified by omitting all unnecessary details. Many of you may have recognized at this point that *AB* and *BC* are *two-force members*. For those of you who have not, we will pursue our analysis, ignoring that fact and assuming that the directions of the reactions at *A* and *C* are unknown. Each of these reactions are represented by two components: $\mathbf{A}_x$ and $\mathbf{A}_y$ at *A*, and $\mathbf{C}_x$ and $\mathbf{C}_y$ at *C*. The equilibrium equations are.

$$+\circlearrowleft \Sigma M_C = 0: \qquad A_x(0.6\text{ m}) - (30\text{ kN})(0.8\text{ m}) = 0$$

$$A_x = +40\text{ kN} \tag{1.1}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \qquad A_x + C_x = 0$$

$$C_x = -A_x \qquad C_x = -40\text{ kN} \tag{1.2}$$

$$+\uparrow \Sigma F_y = 0: \qquad A_y + C_y - 30\text{ kN} = 0$$

$$A_y + C_y = +30\text{ kN} \tag{1.3}$$

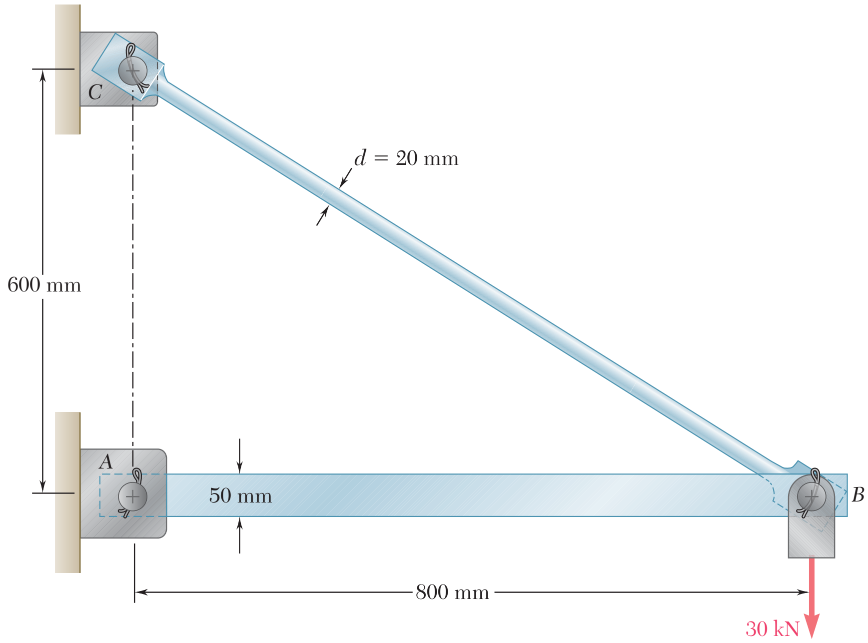


Fig. 1.1 Boom used to support a 30-kN load.

We have found two of the four unknowns, but cannot determine the other two from these equations, and no additional independent equation can be obtained from the free-body diagram of the structure. We must now dismember the structure. Considering the free-body diagram of the boom *AB* (Fig. 1.3), we write the following equilibrium equation:

$$+\circlearrowleft \Sigma M_B = 0: \qquad -A_y(0.8\text{ m}) = 0 \qquad A_y = 0 \tag{1.4}$$

Substituting for A_y from Eq. (1.4) into Eq. (1.3), we obtain $C_y = +30$ kN. Expressing the results obtained for the reactions at *A* and *C* in vector form, we have

$$\mathbf{A} = 40\text{ kN} \rightarrow \qquad \mathbf{C}_x = 40\text{ kN} \leftarrow \qquad \mathbf{C}_y = 30\text{ kN} \uparrow$$

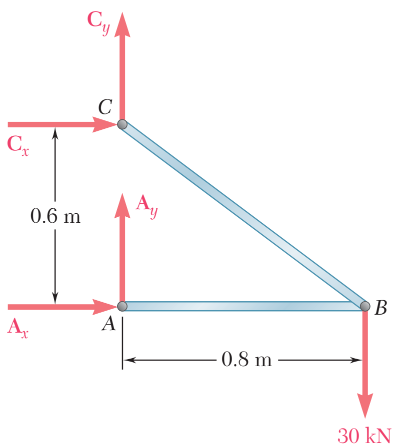


Fig. 1.2 Free-body diagram of boom showing applied load and reaction forces.

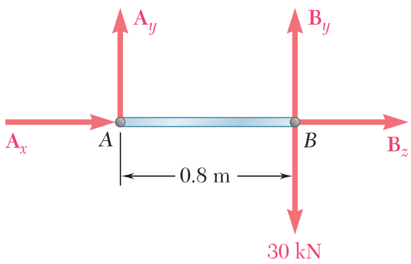


Fig. 1.3 Free-body diagram of member *AB* freed from structure.

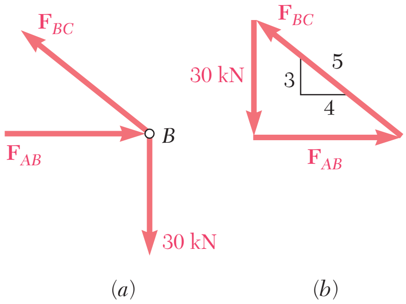


Fig. 1.4 Free-body diagram of boom's joint *B* and associated force triangle.

Note that the reaction at A is directed along the axis of the boom AB and causes compression in that member. Observe that the components C_x and C_y of the reaction at C are, respectively, proportional to the horizontal and vertical components of the distance from B to C and that the reaction at C is equal to 50 kN, is directed along the axis of the rod BC, and causes tension in that member.

These results could have been anticipated by recognizing that AB and BC are two-force members, i.e., members that are subjected to forces at only two points, these points being A and B for member AB, and B and C for member BC. Indeed, for a two-force member the lines of action of the resultants of the forces acting at each of the two points are equal and opposite and pass through both points. Using this property, we could have obtained a simpler solution by considering the free-body diagram of pin B. The forces on pin B, $\mathbf{F}_{AB}$ and $\mathbf{F}_{BC}$, are exerted, respectively, by members AB and BC and the 30-kN load (Fig. 1.4*a*). Pin B is shown to be in equilibrium by drawing the corresponding force triangle (Fig. 1.4*b*).

Since force $\mathbf{F}_{BC}$ is directed along member BC, its slope is the same as that of BC, namely, 3/4. We can, therefore, write the proportion

$$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30 \text{ kN}}{3}$$

from which

$$F_{AB} = 40 \text{ kN} \qquad F_{BC} = 50 \text{ kN}$$

Forces $\mathbf{F}'_{AB}$ and $\mathbf{F}'_{BC}$ exerted by pin B on boom AB and rod BC are equal and opposite to $\mathbf{F}_{AB}$ and $\mathbf{F}_{BC}$ (Fig. 1.5).

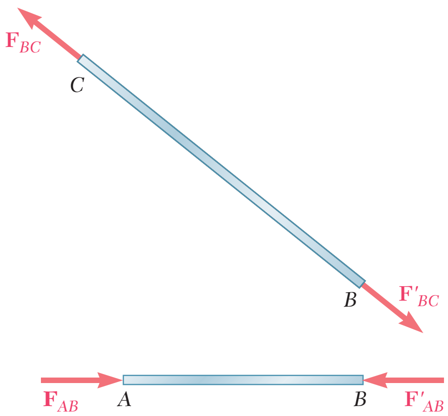


Fig. 1.5 Free-body diagrams of two-force members *AB* and *BC*.

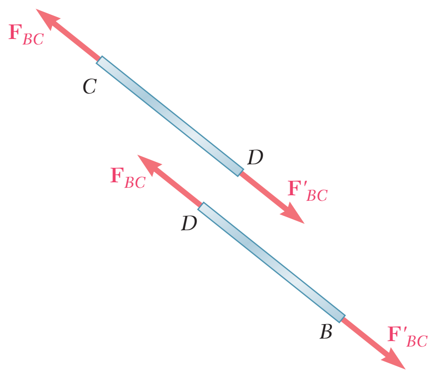


Fig. 1.6 Free-body diagrams of sections of rod *BC*.

Knowing the forces at the ends of each member, we can now determine the internal forces in these members. Passing a section at some arbitrary point D of rod BC, we obtain two portions BD and CD (Fig. 1.6). Since 50-kN forces must be applied at D to both portions of the rod to keep them in equilibrium, an internal force of 50 kN is produced in rod BC when a 30-kN load is applied at B. From the directions of the forces $\mathbf{F}_{BC}$ and $\mathbf{F}'_{BC}$ in Fig. 1.6 we see that the rod is in tension. A similar procedure enables us to determine that the internal force in boom AB is 40 kN and is in compression.

1.2 STRESSES IN THE MEMBERS OF A STRUCTURE

1.2A Axial Stress

In the preceding section, we found forces in individual members. This is the first and necessary step in the analysis of a structure. However it does not tell us whether the given load can be safely supported. Rod BC of the example considered in the preceding section is a two-force member and, therefore, the forces $\mathbf{F}_{BC}$ and $\mathbf{F}'_{BC}$ acting on its ends B and C (Fig. 1.5) are directed along the axis of the rod. Whether rod BC will break or not under this loading depends upon the value found for the internal force F_{BC}, the cross-sectional area of the rod, and the material of which the rod is made. Actually, the internal force F_{BC} represents the resultant of elementary forces distributed over the entire area A of the cross section (Fig. 1.7). The average intensity of these distributed forces is equal to the force per unit area, F_{BC}/A, on the section. Whether or not the rod will break under the given loading depends upon the ability of the material to withstand the corresponding value F_{BC}/A of the intensity of the distributed internal forces.

Photo 1.2 This bridge truss consists of two-force members that may be in tension or in compression.

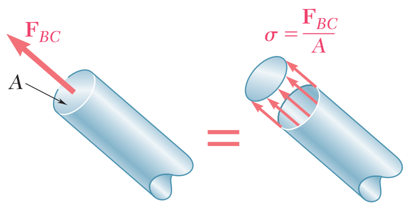


Fig. 1.7 Axial force represents the resultant of distributed elementary forces.

Let us look at the uniformly distributed force using Fig. 1.8. The force per unit area, or intensity of the forces distributed over a given section, is called the *stress* and is denoted by the Greek letter σ (sigma). The stress in a member of cross-sectional area A subjected to an axial load $\mathbf{P}$ is obtained by dividing the magnitude P of the load by the area A:

$$\sigma = \frac{P}{A} \tag{1.5}$$

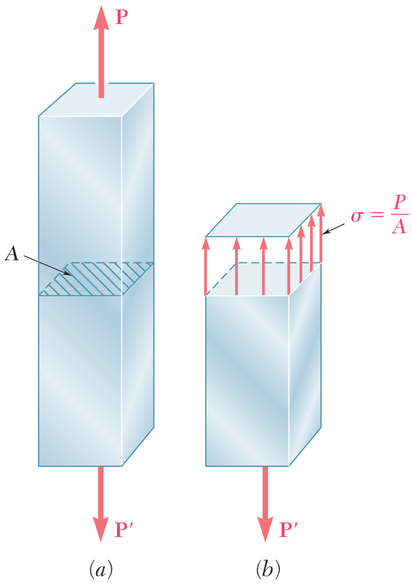


Fig. 1.8 (a) Member with an axial load. (b) Idealized uniform stress distribution at an arbitrary section.

A positive sign indicates a tensile stress (member in tension), and a negative sign indicates a compressive stress (member in compression).

As shown in Fig. 1.8, the section through the rod to determine the internal force in the rod and the corresponding stress is perpendicular to the axis of the rod. The corresponding stress is described as a *normal stress*. Thus, Eq. (1.5) gives the *normal stress in a member under axial loading*:

Note that in Eq. (1.5), σ represents the *average value* of the stress over the cross section, rather than the stress at a specific point of the cross section. To define the stress at a given point Q of the cross section, consider a small area ΔA (Fig. 1.9). Dividing the magnitude of $\Delta\mathbf{F}$ by ΔA, you obtain the average value of the stress over ΔA. Letting ΔA approach zero, the stress at point Q is

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \tag{1.6}$$

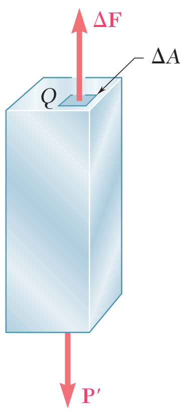


Fig. 1.9 Small area ΔA, at an arbitrary cross section point carries/axial ΔF in this axial member.

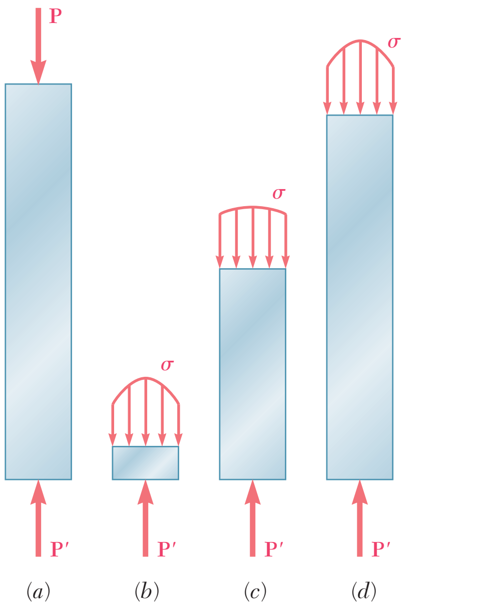


Fig. 1.10 Stress distributions at different sections along axially loaded member.

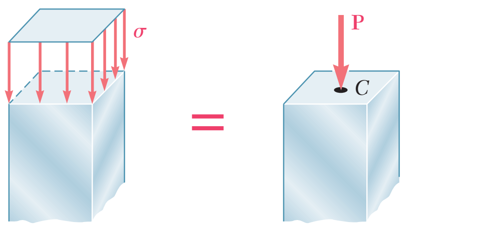


Fig. 1.11 Idealized uniform stress distribution implies the resultant force passes through the cross section's center.

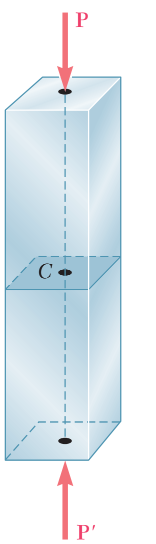


Fig. 1.12 Centric loading having resultant forces passing through the centroid of the section.

In general, the value for the stress σ at a given point Q of the section is different from that for the average stress given by Eq. (1.5), and σ is found to vary across the section. In a slender rod subjected to equal and opposite concentrated loads **P** and **P**′ (Fig. 1.10*a*), this variation is small in a section away from the points of application of the concentrated loads (Fig. 1.10*c*), but it is quite noticeable in the neighborhood of these points (Fig. 1.10*b* and *d*).

It follows from Eq. (1.6) that the magnitude of the resultant of the distributed internal forces is

$$\int dF = \int_A \sigma\, dA$$

But the conditions of equilibrium of each of the portions of rod shown in Fig. 1.10 require that this magnitude be equal to the magnitude P of the concentrated loads. Therefore,

$$P = \int dF = \int_A \sigma\, dA \tag{1.7}$$

which means that the volume under each of the stress surfaces in Fig. 1.10 must be equal to the magnitude P of the loads. However, this is the only information derived from statics regarding the distribution of normal stresses in the various sections of the rod. The actual distribution of stresses in any given section is *statically indeterminate*. To learn more about this distribution, it is necessary to consider the deformations resulting from the particular mode of application of the loads at the ends of the rod. This will be discussed further in Chap. 2.

In practice, it is assumed that the distribution of normal stresses in an axially loaded member is uniform, except in the immediate vicinity of the points of application of the loads. The value σ of the stress is then equal to σ_{ave} and can be obtained from Eq. (1.5). However, realize that when we assume a uniform distribution of stresses in the section, it follows from elementary statics[†] that the resultant **P** of the internal forces must be applied at the centroid C of the section (Fig. 1.11). This means that *a uniform distribution of stress is possible only if the line of action of the concentrated loads* **P** *and* **P**′ *passes through the centroid of the section considered* (Fig. 1.12). This type of loading is called *centric loading* and will take place in all straight two-force members found in trusses and pin-connected structures, such as the one considered in Fig. 1.1. However, if a two-force member is loaded axially, but *eccentrically*, as shown in Fig. 1.13*a*, the conditions of equilibrium of the portion of member in Fig. 1.13*b* show that the internal forces in a given section must be equivalent to a force **P** applied at the centroid of the section and a couple **M** of moment $M = Pd$. This distribution of forces—the corresponding distribution of stresses—*cannot be uniform*. Nor can the distribution of stresses be symmetric. This point will be discussed in detail in Chap. 4.

[†]See Ferdinand P. Beer and E. Russell Johnston, Jr., *Mechanics for Engineers,* 5th ed., McGraw-Hill, New York, 2008, or *Vector Mechanics for Engineers,* 10th ed., McGraw-Hill, New York, 2013, Secs. 5.2 and 5.3.

When SI metric units are used, P is expressed in newtons (N) and A in square meters (m^2), so the stress σ will be expressed in N/m^2. This unit is called a *pascal* (Pa). However, the pascal is an exceedingly small quantity and often multiples of this unit must be used: the kilopascal (kPa), the megapascal (MPa), and the gigapascal (GPa):

$$1 \text{ kPa} = 10^3 \text{ Pa} = 10^3 \text{ N/m}^2$$
$$1 \text{ MPa} = 10^6 \text{ Pa} = 10^6 \text{ N/m}^2$$
$$1 \text{ GPa} = 10^9 \text{ Pa} = 10^9 \text{ N/m}^2$$

When U.S. customary units are used, force P is usually expressed in pounds (lb) or kilopounds (kip), and the cross-sectional area A is given in square inches (in^2). The stress σ then is expressed in pounds per square inch (psi) or kilopounds per square inch (ksi).†

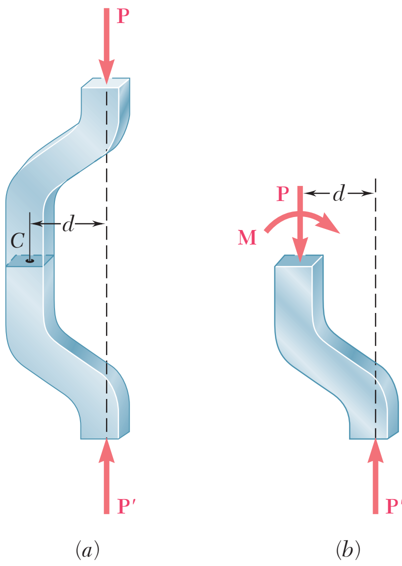


Fig. 1.13 An example of simple eccentric loading.

Concept Application 1.1

Considering the structure of Fig. 1.1 on page 5, assume that rod BC is made of a steel with a maximum allowable stress $\sigma_{all} = 165$ MPa. Can rod BC safely support the load to which it will be subjected? The magnitude of the force F_{BC} in the rod was 50 kN. Recalling that the diameter of the rod is 20 mm, use Eq. (1.5) to determine the stress created in the rod by the given loading.

$$P = F_{BC} = +50 \text{ kN} = +50 \times 10^3 \text{ N}$$

$$A = \pi r^2 = \pi\left(\frac{20 \text{ mm}}{2}\right)^2 = \pi(10 \times 10^{-3} \text{ m})^2 = 314 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{+50 \times 10^3 \text{ N}}{314 \times 10^{-6} \text{ m}^2} = +159 \times 10^6 \text{ Pa} = +159 \text{ MPa}$$

Since σ is smaller than σ_{all} of the allowable stress in the steel used, rod BC can safely support the load.

To be complete, our analysis of the given structure should also include the compressive stress in boom AB, as well as the stresses produced in the pins and their bearings. This will be discussed later in this chapter. You should also determine whether the deformations produced by the given loading are acceptable. The study of deformations under axial loads will be the subject of Chap. 2. For members in compression, the *stability* of the member (i.e., its ability to support a given load without experiencing a sudden change in configuration) will be discussed in Chap. 10.

†The principal SI and U.S. Customary units used in mechanics are listed in tables inside the front cover of this book. From the table on the right-hand side, 1 psi is approximately equal to 7 kPa and 1 ksi approximately equal to 7 MPa.

The engineer's role is not limited to the analysis of existing structures and machines subjected to given loading conditions. Of even greater importance is the *design* of new structures and machines, that is the selection of appropriate components to perform a given task.

Concept Application 1.2

As an example of design, let us return to the structure of Fig. 1.1 on page 5 and assume that aluminum with an allowable stress $\sigma_{all} = 100$ MPa is to be used. Since the force in rod BC is still $P = F_{BC} = 50$ kN under the given loading, from Eq. (1.5), we have

$$\sigma_{all} = \frac{P}{A} \qquad A = \frac{P}{\sigma_{all}} = \frac{50 \times 10^3 \text{ N}}{100 \times 10^6 \text{ Pa}} = 500 \times 10^{-6} \text{ m}^2$$

and since $A = \pi r^2$,

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{500 \times 10^{-6} \text{ m}^2}{\pi}} = 12.62 \times 10^{-3} \text{ m} = 12.62 \text{ mm}$$

$$d = 2r = 25.2 \text{ mm}$$

Therefore, an aluminum rod 26 mm or more in diameter will be adequate.

1.2B Shearing Stress

The internal forces and the corresponding stresses discussed in Sec. 1.2A were normal to the section considered. A very different type of stress is obtained when transverse forces **P** and **P**′ are applied to a member AB (Fig. 1.14). Passing a section at C between the points of application of the two forces (Fig. 1.15a), you obtain the diagram of portion AC shown in

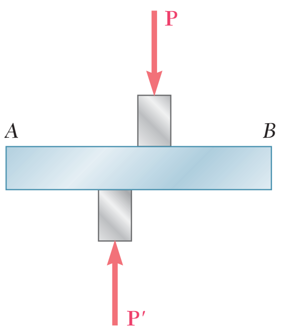


Fig. 1.14 Opposing transverse loads creating shear on member *AB*.

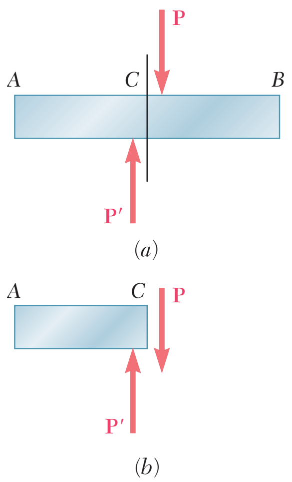


Fig. 1.15 This shows the resulting internal shear force on a section between transverse forces.

Fig. 1.15*b*. Internal forces must exist in the plane of the section, and their resultant is equal to **P**. These elementary internal forces are called *shearing forces,* and the magnitude P of their resultant is the *shear* in the section. Dividing the shear P by the area A of the cross section, you obtain the *average shearing stress* in the section. Denoting the shearing stress by the Greek letter τ (tau), write

$$\tau_{\text{ave}} = \frac{P}{A} \tag{1.8}$$

The value obtained is an average value of the shearing stress over the entire section. Contrary to what was said earlier for normal stresses, the distribution of shearing stresses across the section *cannot* be assumed to be uniform. As you will see in Chap. 6, the actual value τ of the shearing stress varies from zero at the surface of the member to a maximum value τ_{max} that may be much larger than the average value τ_{ave}.

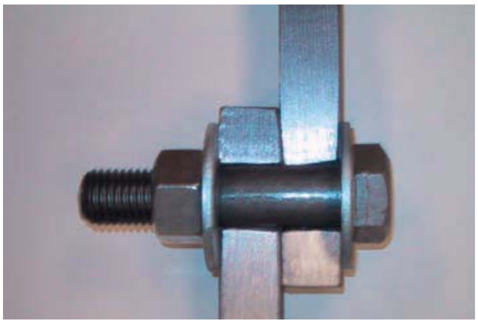


Photo 1.3 Cutaway view of a connection with a bolt in shear.

Shearing stresses are commonly found in bolts, pins, and rivets used to connect various structural members and machine components (Photo 1.3). Consider the two plates A and B, which are connected by a bolt CD (Fig. 1.16). If the plates are subjected to tension forces of magnitude F, stresses will develop in the section of bolt corresponding to the plane EE'. Drawing the diagrams of the bolt and of the portion located above the plane EE' (Fig. 1.17), the shear P in the section is equal to F. The average shearing stress in the section is obtained using Eq. (1.8) by dividing the shear $P = F$ by the area A of the cross section:

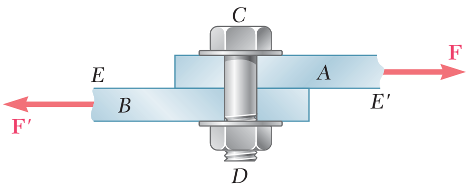


Fig. 1.16 Bolt subject to single shear.

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{A} \tag{1.9}$$

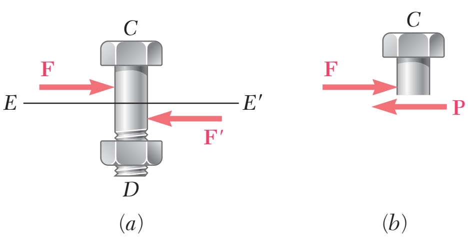


Fig. 1.17 (*a*) Diagram of bolt in single shear; (*b*) section *E-E′* of the bolt.

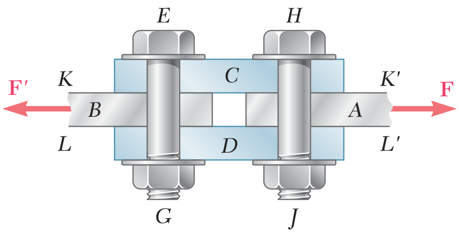


Fig. 1.18 Bolts subject to double shear.

The previous bolt is said to be in *single shear*. Different loading situations may arise, however. For example, if splice plates C and D are used to connect plates A and B (Fig. 1.18), shear will take place in bolt HJ in each of the two planes KK' and LL' (and similarly in bolt EG). The bolts are said to be in *double shear*. To determine the average shearing stress in each plane, draw free-body diagrams of bolt HJ and of the portion of the bolt located between the two planes (Fig. 1.19). Observing that the shear P in each of the sections is $P = F/2$, the average shearing stress is

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F/2}{A} = \frac{F}{2A} \tag{1.10}$$

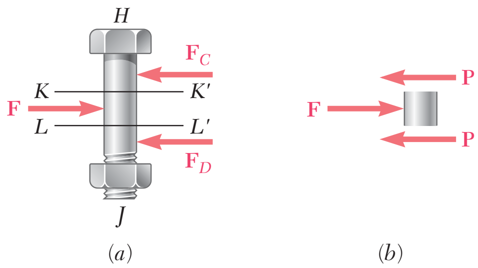


Fig. 1.19 (a) Diagram of bolt in double shear; (b) section *K-K′* and *L-L′* of the bolt.

1.2C Bearing Stress in Connections

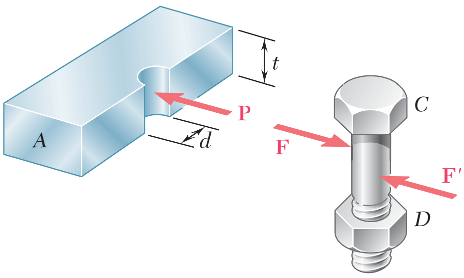


Fig. 1.20 Equal and opposite forces between plate and bolt, exerted over bearing surfaces.

Bolts, pins, and rivets create stresses in the members they connect along the *bearing surface* or surface of contact. For example, consider again the two plates A and B connected by a bolt CD that were discussed in the preceding section (Fig. 1.16). The bolt exerts on plate A a force $\mathbf{P}$ equal and opposite to the force $\mathbf{F}$ exerted by the plate on the bolt (Fig. 1.20). The force $\mathbf{P}$ represents the resultant of elementary forces distributed on the inside surface of a half-cylinder of diameter d and of length t equal to the thickness of the plate. Since the distribution of these forces—and of the corresponding stresses—is quite complicated, in practice one uses an average nominal value σ_b of the stress, called the *bearing stress,* which is obtained by dividing the load P by the area of the rectangle representing the projection of the bolt on the plate section (Fig. 1.21). Since this area is equal to td, where t is the plate thickness and d the diameter of the bolt, we have

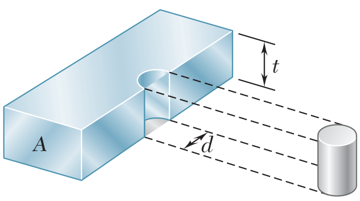


Fig. 1.21 Dimensions for calculating bearing stress area.

$$\sigma_b = \frac{P}{A} = \frac{P}{td} \tag{1.11}$$

1.2D Application to the Analysis and Design of Simple Structures

We are now in a position to determine the stresses in the members and connections of various simple two-dimensional structures and to design such structures. This is illustrated through the following Concept Application.

Concept Application 1.3

Returning to the structure of Fig. 1.1, we will determine the normal stresses, shearing stresses and bearing stresses. As shown in Fig. 1.22, the 20-mm-diameter rod BC has flat ends of 20×40-mm rectangular cross section, while boom AB has a 30×50-mm rectangular cross section and is fitted with a clevis at end B. Both members are connected at B by a pin from which the 30-kN load is suspended by means of a U-shaped bracket. Boom AB is supported at A by a pin fitted into a double bracket, while rod BC is connected at C to a single bracket. All pins are 25 mm in diameter.

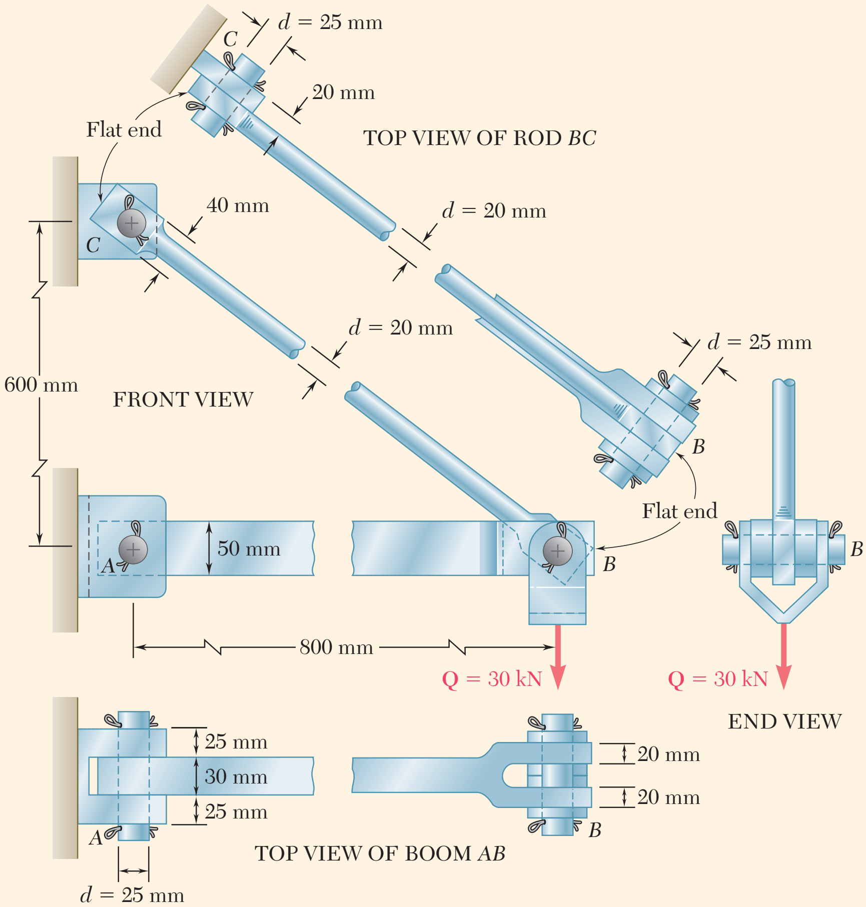


Fig. 1.22 Components of boom used to support 30 kN load.

Normal Stress in Boom *AB* and Rod *BC*. As found in Sec. 1.1A, the force in rod BC is $F_{BC} = 50$ kN (tension) and the area of its circular cross section is $A = 314 \times 10^{-6}$ m^2. The corresponding average normal stress is $\sigma_{BC} = +159$ MPa. However, the flat parts of the rod are also under tension and at the narrowest section. Where the hole is located, we have

$$A = (20 \text{ mm})(40 \text{ mm} - 25 \text{ mm}) = 300 \times 10^{-6} \text{ m}^2$$

(continued)

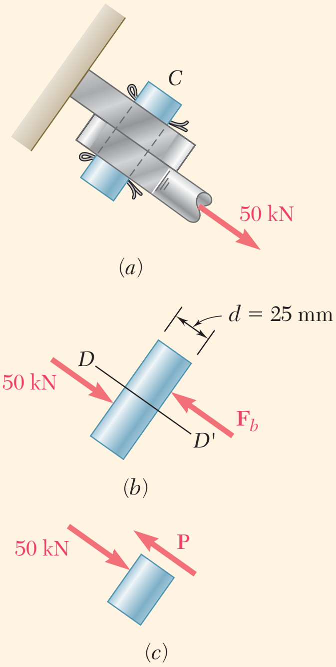


Fig. 1.23 Diagrams of the single shear pin at *C*.

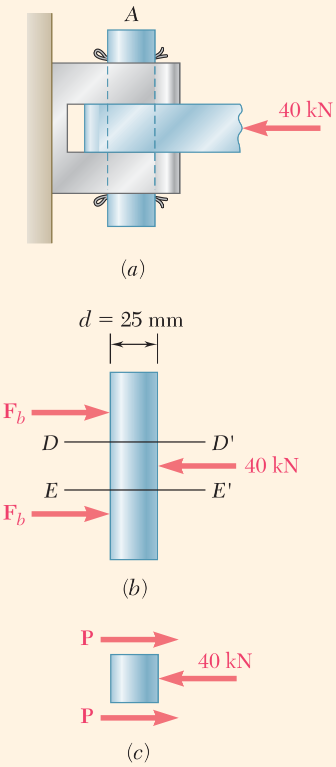


Fig. 1.24 Free-body diagrams of the double shear pin at *A*.

The corresponding average value of the stress is

$$(\sigma_{BC})_{\text{end}} = \frac{P}{A} = \frac{50 \times 10^3 \text{ N}}{300 \times 10^{-6} \text{ m}^2} = 167.0 \text{ MPa}$$

Note that this is an *average value*. Close to the hole the stress will actually reach a much larger value, as you will see in Sec. 2.11. Under an increasing load, the rod will fail near one of the holes rather than in its cylindrical portion; its design could be improved by increasing the width or the thickness of the flat ends of the rod.

Recall from Sec. 1.1A that the force in boom AB is $F_{AB} = 40$ kN (compression). Since the area of the boom's rectangular cross section is $A = 30 \text{ mm} \times 50 \text{ mm} = 1.5 \times 10^{-3} \text{ m}^2$, the average value of the normal stress in the main part of the rod between pins A and B is

$$\sigma_{AB} = -\frac{40 \times 10^3 \text{ N}}{1.5 \times 10^{-3} \text{ m}^2} = -26.7 \times 10^6 \text{ Pa} = -26.7 \text{ MPa}$$

Note that the sections of minimum area at A and B are not under stress, since the boom is in compression, and therefore *pushes* on the pins (instead of *pulling* on the pins as rod BC does).

Shearing Stress in Various Connections. To determine the shearing stress in a connection such as a bolt, pin, or rivet, you first show the forces exerted by the various members it connects. In the case of pin C (Fig. 1.23*a*), draw Fig. 1.23*b* to show the 50-kN force exerted by member BC on the pin, and the equal and opposite force exerted by the bracket. Drawing the diagram of the portion of the pin located below the plane DD' where shearing stresses occur (Fig. 1.23*c*), notice that the shear in that plane is $P = 50$ kN. Since the cross-sectional area of the pin is

$$A = \pi r^2 = \pi\left(\frac{25 \text{ mm}}{2}\right)^2 = \pi(12.5 \times 10^{-3} \text{ m})^2 = 491 \times 10^{-6} \text{ m}^2$$

the average value of the shearing stress in the pin at C is

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{50 \times 10^3 \text{ N}}{491 \times 10^{-6} \text{ m}^2} = 102.0 \text{ MPa}$$

Note that pin A (Fig. 1.24) is in double shear. Drawing the free-body diagrams of the pin and the portion of pin located between the planes DD' and EE' where shearing stresses occur, we see that $P = 20$ kN and

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{20 \text{ kN}}{491 \times 10^{-6} \text{ m}^2} = 40.7 \text{ MPa}$$

Pin B (Fig. 1.25*a*) can be divided into five portions that are acted upon by forces exerted by the boom, rod, and bracket. Portions DE (Fig. 1.25*b*) and DG (Fig. 1.25*c*) show that the shear in section E is $P_E = 15$ kN and the shear in section G is $P_G = 25$ kN. Since the loading

(continued)

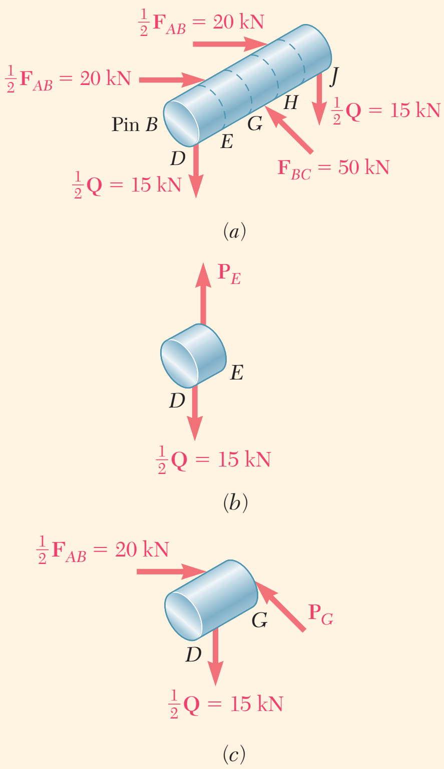


Fig. 1.25 Free-body diagrams for various sections at pin *B*.

of the pin is symmetric, the maximum value of the shear in pin *B* is $P_G = 25$ kN, and the largest the shearing stresses occur in sections *G* and *H*, where

$$\tau_{\text{ave}} = \frac{P_G}{A} = \frac{25 \text{ kN}}{491 \times 10^{-6} \text{ m}^2} = 50.9 \text{ MPa}$$

Bearing Stresses. Use Eq. (1.11) to determine the nominal bearing stress at *A* in member *AB*. From Fig. 1.22, $t = 30$ mm and $d = 25$ mm. Recalling that $P = F_{AB} = 40$ kN, we have

$$\sigma_b = \frac{P}{td} = \frac{40 \text{ kN}}{(30 \text{ mm})(25 \text{ mm})} = 53.3 \text{ MPa}$$

To obtain the bearing stress in the bracket at *A*, use $t = 2(25 \text{ mm}) = 50$ mm and $d = 25$ mm:

$$\sigma_b = \frac{P}{td} = \frac{40 \text{ kN}}{(50 \text{ mm})(25 \text{ mm})} = 32.0 \text{ MPa}$$

The bearing stresses at *B* in member *AB*, at *B* and *C* in member *BC*, and in the bracket at *C* are found in a similar way.

1.2E Method of Problem Solution

You should approach a problem in mechanics as you would approach an actual engineering situation. By drawing on your own experience and intuition about physical behavior, you will find it easier to understand and formulate the problem. Your solution must be based on the fundamental principles of statics and on the principles you will learn in this text. Every step you take in the solution must be justified on this basis, leaving no room for your intuition or "feeling." After you have obtained an answer, you should check it. Here again, you may call upon your common sense and personal experience. If you are not completely satisfied with the result, you should carefully check your formulation of the problem, the validity of the methods used for its solution, and the accuracy of your computations.

In general, you can usually solve problems in several different ways; there is no one approach that works best for everybody. However, we have found that students often find it helpful to have a general set of guidelines to use for framing problems and planning solutions. In the Sample Problems throughout this text, we use a four-step approach for solving problems, which we refer to as the SMART methodology: Strategy, Modeling, Analysis, and Reflect & Think:

1. **Strategy.** The statement of a problem should be clear and precise, and should contain the given data and indicate what information is required. The first step in solving the problem is to decide what concepts you have learned that apply to the given situation and

connect the data to the required information. It is often useful to work backward from the information you are trying to find: ask yourself what quantities you need to know to obtain the answer, and if some of these quantities are unknown, how can you find them from the given data.

2. **Modeling.** The solution of most problems encountered will require that you first determine the *reactions at the supports* and *internal forces and couples*. It is important to include one or several *free-body diagrams* to support these determinations. Draw additional sketches as necessary to guide the remainder of your solution, such as for stress analyses.
3. **Analysis.** After you have drawn the appropriate diagrams, use the fundamental principles of mechanics to write equilibrium equations. These equations can be solved for unknown forces and used to compute the required stresses and deformations.
4. **Reflect & Think.** After you have obtained the answer, check it carefully. Does it make sense in the context of the original problem? You can often detect mistakes in *reasoning* by carrying the units through your computations and checking the units obtained for the answer. For example, in the design of the rod discussed in Concept Application 1.2, the required diameter of the rod was expressed in millimeters, which is the correct unit for a dimension; if you had obtained another unit, you would know that some mistake had been made.

You can often detect errors in *computation* by substituting the numerical answer into an equation that was not used in the solution and verifying that the equation is satisfied. The importance of correct computations in engineering cannot be overemphasized.

Numerical Accuracy. The accuracy of the solution of a problem depends upon two items: (1) the accuracy of the given data and (2) the accuracy of the computations performed.

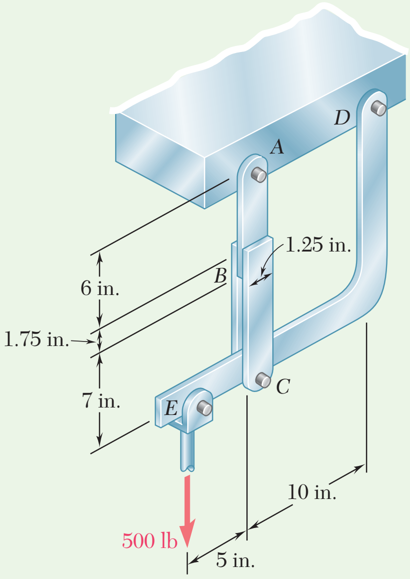
The solution cannot be more accurate than the less accurate of these two items. For example, if the loading of a beam is known to be 75,000 lb with a possible error of 100 lb either way, the relative error that measures the degree of accuracy of the data is

$$\frac{100 \text{ lb}}{75{,}000 \text{ lb}} = 0.0013 = 0.13\%$$

To compute the reaction at one of the beam supports, it would be meaningless to record it as 14,322 lb. The accuracy of the solution cannot be greater than 0.13%, no matter how accurate the computations are, and the possible error in the answer may be as large as $(0.13/100)(14{,}322 \text{ lb}) \approx 20$ lb. The answer should be properly recorded as $14{,}320 \pm 20$ lb.

In engineering problems, the data are seldom known with an accuracy greater than 0.2%. A practical rule is to use four figures to record numbers beginning with a "1" and three figures in all other cases. Unless otherwise indicated, the data given are assumed to be known with a comparable degree of accuracy. A force of 40 lb, for example, should be read 40.0 lb, and a force of 15 lb should be read 15.00 lb.

The speed and accuracy of calculators and computers makes the numerical computations in the solution of many problems much easier. However, students should not record more significant figures than can be justified merely because they are easily obtained. An accuracy greater than 0.2% is seldom necessary or meaningful in the solution of practical engineering problems.



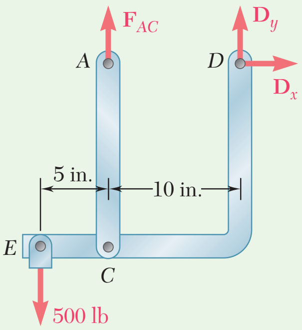


Fig. 1 Free-body diagram of hanger.

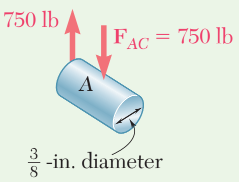


Fig. 2 Pin *A*.

Sample Problem 1.1

In the hanger shown, the upper portion of link *ABC* is $\frac{3}{8}$ in. thick and the lower portions are each $\frac{1}{4}$ in. thick. Epoxy resin is used to bond the upper and lower portions together at *B*. The pin at *A* has a $\frac{3}{8}$-in. diameter, while a $\frac{1}{4}$-in.-diameter pin is used at *C*. Determine (*a*) the shearing stress in pin *A*, (*b*) the shearing stress in pin *C*, (*c*) the largest normal stress in link *ABC*, (*d*) the average shearing stress on the bonded surfaces at *B*, and (*e*) the bearing stress in the link at *C*.

STRATEGY: Consider the free body of the hanger to determine the internal force for member *AB* and then proceed to determine the shearing and bearing forces applicable to the pins. These forces can then be used to determine the stresses.

MODELING: Draw the free-body diagram of the hanger to determine the support reactions (Fig. 1). Then draw the diagrams of the various components of interest showing the forces needed to determine the desired stresses (Figs. 2-6).

ANALYSIS:

Free Body: Entire Hanger. Since the link *ABC* is a two-force member (Fig. 1), the reaction at *A* is vertical; the reaction at *D* is represented by its components D_x and D_y. Thus,

$$+\circlearrowleft \Sigma M_D = 0: \qquad (500 \text{ lb})(15 \text{ in.}) - F_{AC}(10 \text{ in.}) = 0$$

$$F_{AC} = +750 \text{ lb} \qquad F_{AC} = 750 \text{ lb} \qquad \textit{tension}$$

a. Shearing Stress in Pin *A*. Since this $\frac{3}{8}$-in.-diameter pin is in single shear (Fig. 2), write

$$\tau_A = \frac{F_{AC}}{A} = \frac{750 \text{ lb}}{\frac{1}{4}\pi(0.375 \text{ in.})^2} \qquad \tau_A = 6790 \text{ psi} \blacktriangleleft$$

b. Shearing Stress in Pin *C*. Since this $\frac{1}{4}$-in.-diameter pin is in double shear (Fig. 3), write

$$\tau_C = \frac{\frac{1}{2}F_{AC}}{A} = \frac{375 \text{ lb}}{\frac{1}{4}\pi(0.25 \text{ in.})^2} \qquad \tau_C = 7640 \text{ psi} \blacktriangleleft$$

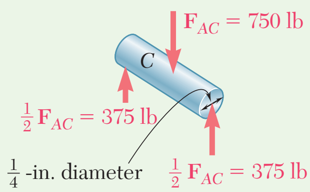


Fig. 3 Pin *C*.

(continued)

c. Largest Normal Stress in Link *ABC*. The largest stress is found where the area is smallest; this occurs at the cross section at A (Fig. 4) where the $\frac{3}{8}$-in. hole is located. We have

$$\sigma_A = \frac{F_{AC}}{A_{\text{net}}} = \frac{750 \text{ lb}}{(\frac{3}{8}\text{ in.})(1.25 \text{ in.} - 0.375 \text{ in.})} = \frac{750 \text{ lb}}{0.328 \text{ in}^2} \qquad \sigma_A = 2290 \text{ psi} \blacktriangleleft$$

d. Average Shearing Stress at *B*. We note that bonding exists on both sides of the upper portion of the link (Fig. 5) and that the shear force on each side is $F_1 = (750 \text{ lb})/2 = 375$ lb. The average shearing stress on each surface is

$$\tau_B = \frac{F_1}{A} = \frac{375 \text{ lb}}{(1.25 \text{ in.})(1.75 \text{ in.})} \qquad \tau_B = 171.4 \text{ psi} \blacktriangleleft$$

e. Bearing Stress in Link at *C*. For each portion of the link (Fig. 6), $F_1 = 375$ lb, and the nominal bearing area is (0.25 in.)(0.25 in.) $= 0.0625 \text{ in}^2$.

$$\sigma_b = \frac{F_1}{A} = \frac{375 \text{ lb}}{0.0625 \text{ in}^2} \qquad \sigma_b = 6000 \text{ psi} \blacktriangleleft$$

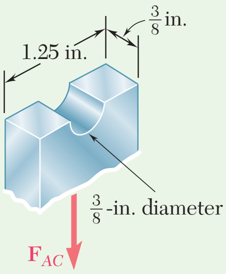


Fig. 4 Link *ABC* section at *A*.

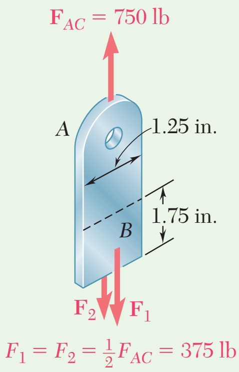


Fig. 5 Element *AB*.

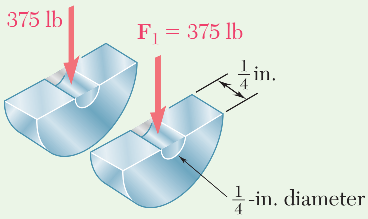
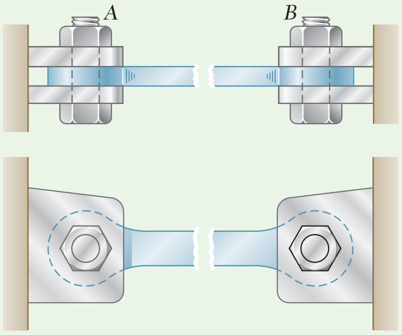


Fig. 6 Link *ABC* section at *C*.

REFLECT and THINK: This sample problem demonstrates the need to draw free-body diagrams of the separate components, carefully considering the behavior in each one. As an example, based on visual inspection of the hanger it is apparent that member AC should be in tension for the given load, and the analysis confirms this. Had a compression result been obtained instead, a thorough reexamination of the analysis would have been required.

Sample Problem 1.2



The steel tie bar shown is to be designed to carry a tension force of magnitude $P = 120$ kN when bolted between double brackets at A and B. The bar will be fabricated from 20-mm-thick plate stock. For the grade of steel to be used, the maximum allowable stresses are $\sigma = 175$ MPa, $\tau = 100$ MPa, and $\sigma_b = 350$ MPa. Design the tie bar by determining the required values of (a) the diameter d of the bolt, (b) the dimension b at each end of the bar, and (c) the dimension h of the bar.

STRATEGY: Use free-body diagrams to determine the forces needed to obtain the stresses in terms of the design tension force. Setting these stresses equal to the allowable stresses provides for the determination of the required dimensions.

MODELING and ANALYSIS:

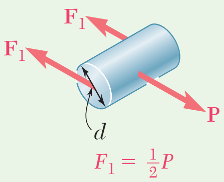


Fig. 1 Sectioned bolt.

a. Diameter of the Bolt. Since the bolt is in double shear (Fig. 1), $F_1 = \frac{1}{2}P = 60$ kN.

$$\tau = \frac{F_1}{A} = \frac{60 \text{ kN}}{\frac{1}{4}\pi\, d^2} \qquad 100 \text{ MPa} = \frac{60 \text{ kN}}{\frac{1}{4}\pi\, d^2} \qquad d = 27.6 \text{ mm}$$

Use **$d = 28$ mm** ◀

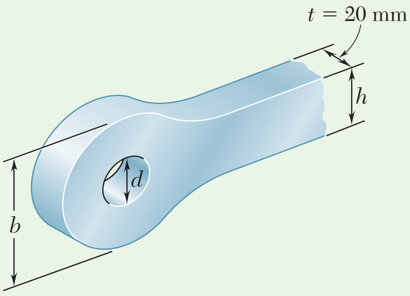


Fig. 2 Tie bar geometry.

At this point, check the bearing stress between the 20-mm-thick plate (Fig. 2) and the 28-mm-diameter bolt.

$$\sigma_b = \frac{P}{td} = \frac{120 \text{ kN}}{(0.020 \text{ m})(0.028 \text{ m})} = 214 \text{ MPa} < 350 \text{ MPa} \qquad \text{OK}$$

b. Dimension b at Each End of the Bar. We consider one of the end portions of the bar in Fig. 3. Recalling that the thickness of the steel plate is $t = 20$ mm and that the average tensile stress must not exceed 175 MPa, write

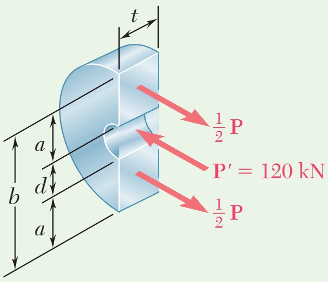


Fig. 3 End section of tie bar.

$$\sigma = \frac{\frac{1}{2}P}{ta} \qquad 175 \text{ MPa} = \frac{60 \text{ kN}}{(0.02 \text{ m})a} \qquad a = 17.14 \text{ mm}$$

$$b = d + 2a = 28 \text{ mm} + 2(17.14 \text{ mm}) \qquad \mathbf{b = 62.3 \text{ mm}}$$ ◀

c. Dimension h of the Bar. We consider a section in the central portion of the bar (Fig. 4). Recalling that the thickness of the steel plate is $t = 20$ mm, we have

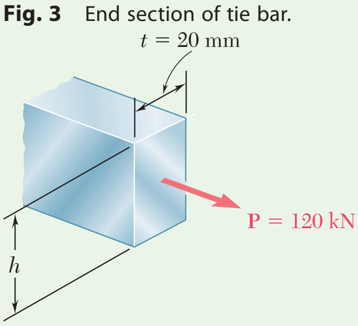


Fig. 4 Mid-body section of tie bar.

$$\sigma = \frac{P}{th} \qquad 175 \text{ MPa} = \frac{120 \text{ kN}}{(0.020 \text{ m})h} \qquad h = 34.3 \text{ mm}$$

Use **$h = 35$ mm** ◀

REFLECT and THINK: We sized d based on bolt shear, and then checked bearing on the tie bar. Had the maximum allowable bearing stress been exceeded, we would have had to recalculate d based on the bearing criterion.

Problems

1.1 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that $d_1 = 30$ mm and $d_2 = 50$ mm, find the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

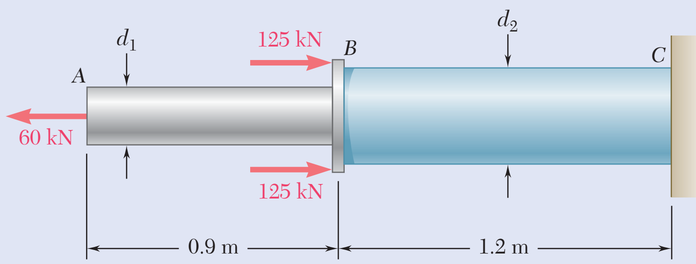


Fig. P1.1 and P1.2

1.2 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that the average normal stress must not exceed 150 MPa in either rod, determine the smallest allowable values of the diameters d_1 and d_2.

1.3 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that $P = 10$ kips, find the average normal stress at the midsection of (*a*) rod *AB,* (*b*) rod *BC.*

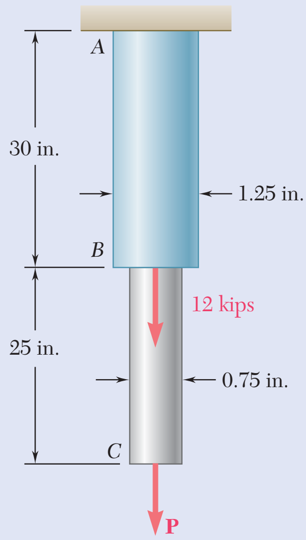


Fig. P1.3 and P1.4

1.4 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Determine the magnitude of the force **P** for which the tensile stresses in rods *AB* and *BC* are equal.

1.5 A strain gage located at C on the surface of bone AB indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200-N forces as shown. Assuming the cross section of the bone at C to be annular and knowing that its outer diameter is 25 mm, determine the inner diameter of the bone's cross section at C.

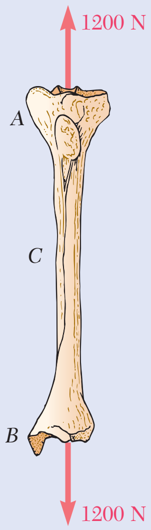


Fig. P1.5

1.6 Two brass rods AB and BC, each of uniform diameter, will be brazed together at B to form a nonuniform rod of total length 100 m that will be suspended from a support at A as shown. Knowing that the density of brass is 8470 kg/m^3, determine (*a*) the length of rod AB for which the maximum normal stress in ABC is minimum, (*b*) the corresponding value of the maximum normal stress.

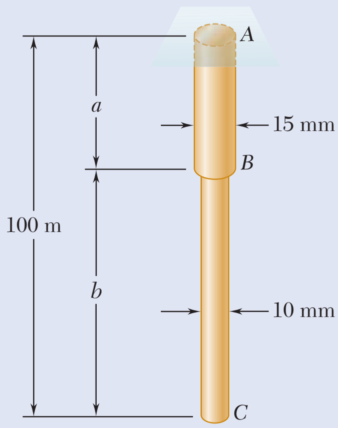


Fig. P1.6

1.7 Each of the four vertical links has an 8×36-mm uniform rectangular cross section, and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points B and D, (*b*) points C and E.

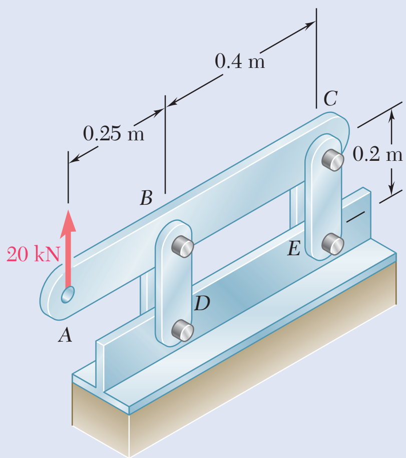


Fig. P1.7

1.8 Link AC has a uniform rectangular cross section $\frac{1}{8}$ in. thick and 1 in. wide. Determine the normal stress in the central portion of the link.

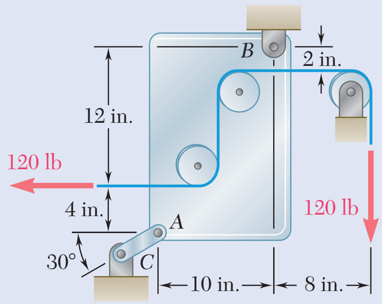


Fig. P1.8

1.9 Three forces, each of magnitude P = 4 kN, are applied to the structure shown. Determine the cross-sectional area of the uniform portion of rod BE for which the normal stress in that portion is +100 MPa.

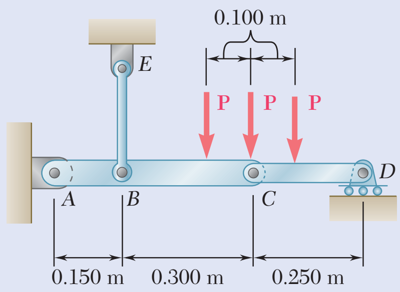


Fig. P1.9

1.10 Link BD consists of a single bar 1 in. wide and $\frac{1}{2}$ in. thick. Knowing that each pin has a $\frac{3}{8}$-in. diameter, determine the maximum value of the average normal stress in link BD if (*a*) $\theta = 0$, (*b*) $\theta = 90°$.

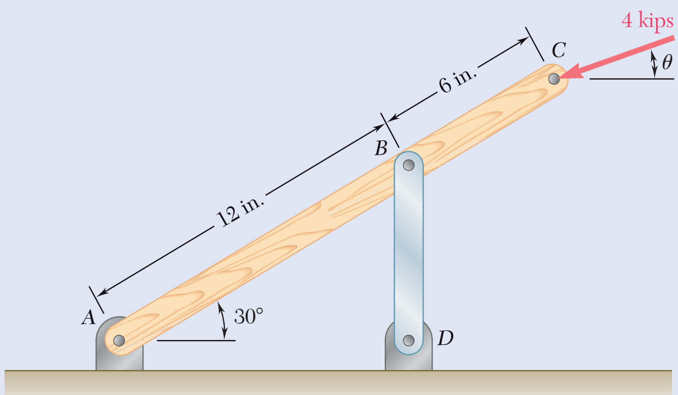


Fig. P1.10

1.11 For the Pratt bridge truss and loading shown, determine the average normal stress in member BE, knowing that the cross-sectional area of that member is 5.87 in^2.

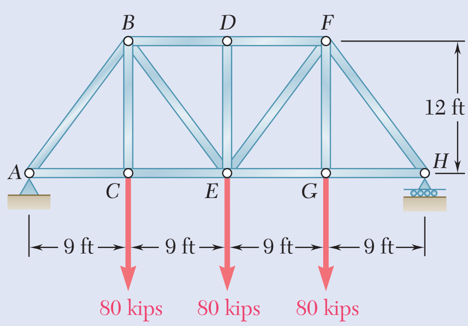


Fig. *P1.11*

1.12 The frame shown consists of *four* wooden members, *ABC*, *DEF*, *BE*, and *CF*. Knowing that each member has a 2 × 4-in. rectangular cross section and that each pin has a $\frac{1}{2}$-in. diameter, determine the maximum value of the average normal stress (*a*) in member *BE*, (*b*) in member *CF*.

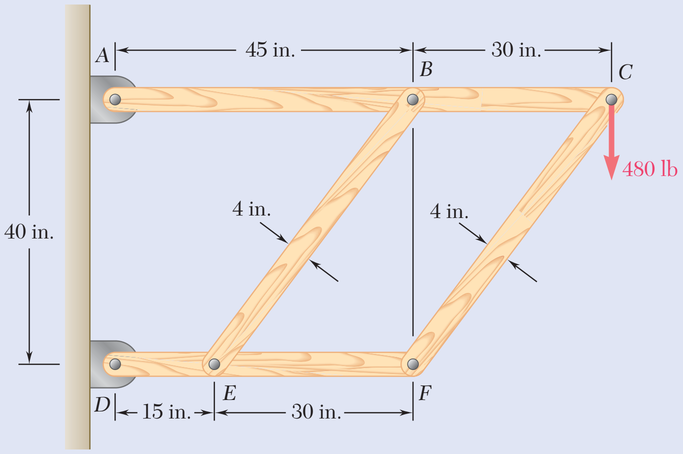


Fig. ***P1.12***

1.13 An aircraft tow bar is positioned by means of a single hydraulic cylinder connected by a 25-mm-diameter steel rod to two identical arm-and-wheel units *DEF*. The mass of the entire tow bar is 200 kg, and its center of gravity is located at *G*. For the position shown, determine the normal stress in the rod.

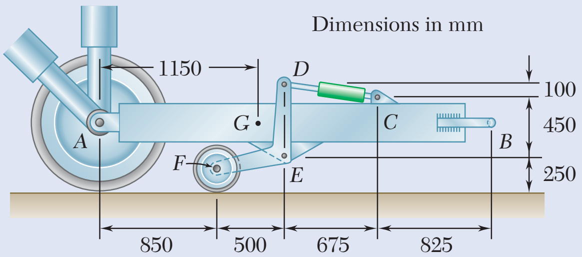


Fig. P1.13

1.14 Two hydraulic cylinders are used to control the position of the robotic arm *ABC*. Knowing that the control rods attached at *A* and *D* each have a 20-mm diameter and happen to be parallel in the position shown, determine the average normal stress in (*a*) member *AE*, (*b*) member *DG*.

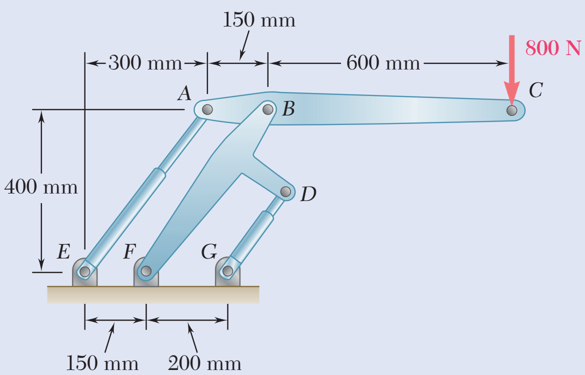


Fig. P1.14

1.15 Determine the diameter of the largest circular hole that can be punched into a sheet of polystyrene 6 mm thick, knowing that the force exerted by the punch is 45 kN and that a 55-MPa average shearing stress is required to cause the material to fail.

1.16 Two wooden planks, each $\frac{1}{2}$ in. thick and 9 in. wide, are joined by the dry mortise joint shown. Knowing that the wood used shears off along its grain when the average shearing stress reaches 1.20 ksi, determine the magnitude P of the axial load that will cause the joint to fail.

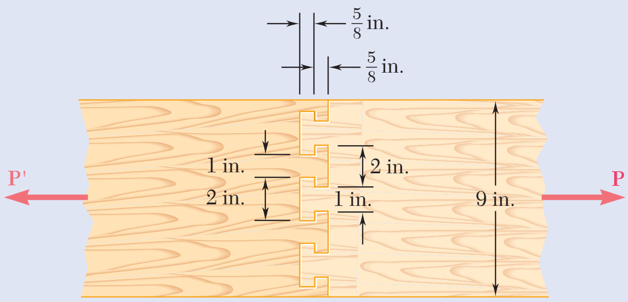


Fig. P1.16

1.17 When the force **P** reached 1600 lb, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.

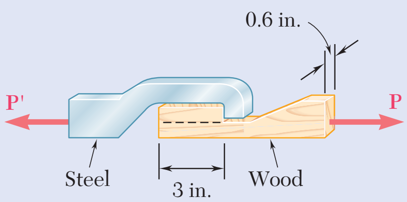


Fig. P1.17

1.18 A load **P** is applied to a steel rod supported as shown by an aluminum plate into which a 12-mm-diameter hole has been drilled. Knowing that the shearing stress must not exceed 180 MPa in the steel rod and 70 MPa in the aluminum plate, determine the largest load **P** that can be applied to the rod.

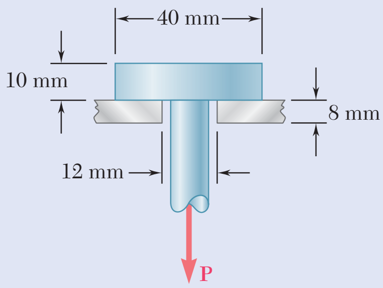


Fig. P1.18

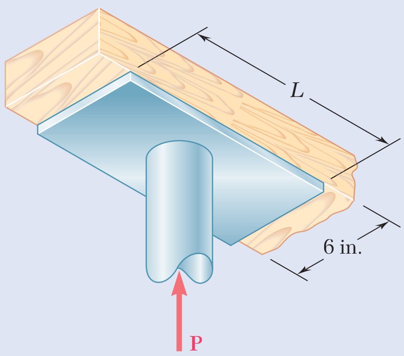


Fig. *P1.19*

1.19 The axial force in the column supporting the timber beam shown is $P = 20$ kips. Determine the smallest allowable length L of the bearing plate if the bearing stress in the timber is not to exceed 400 psi.

1.20 Three wooden planks are fastened together by a series of bolts to form a column. The diameter of each bolt is 12 mm and the inner diameter of each washer is 16 mm, which is slightly larger than the diameter of the holes in the planks. Determine the smallest allowable outer diameter d of the washers, knowing that the average normal stress in the bolts is 36 MPa and that the bearing stress between the washers and the planks must not exceed 8.5 MPa.

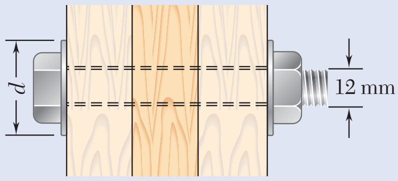


Fig. P1.20

1.21 A 40-kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine (*a*) the maximum bearing stress on the concrete footing, (*b*) the size of the footing for which the average bearing stress in the soil is 145 kPa.

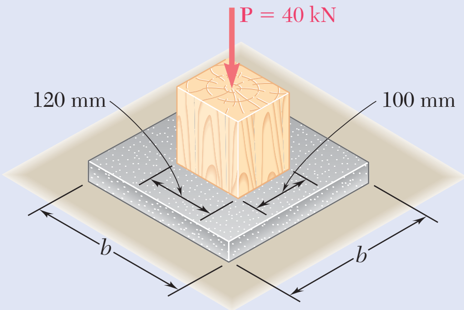


Fig. P1.21

1.22 An axial load **P** is supported by a short W8 × 40 column of cross-sectional area $A = 11.7 \text{ in}^2$ and is distributed to a concrete foundation by a square plate as shown. Knowing that the average normal stress in the column must not exceed 30 ksi and that the bearing stress on the concrete foundation must not exceed 3.0 ksi, determine the side a of the plate that will provide the most economical and safe design.

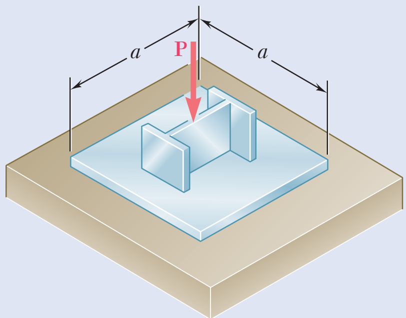


Fig. *P1.22*

1.23 Link AB, of width $b = 2$ in. and thickness $t = \frac{1}{4}$ in., is used to support the end of a horizontal beam. Knowing that the average normal stress in the link is −20 ksi and that the average shearing stress in each of the two pins is 12 ksi determine (*a*) the diameter d of the pins, (*b*) the average bearing stress in the link.

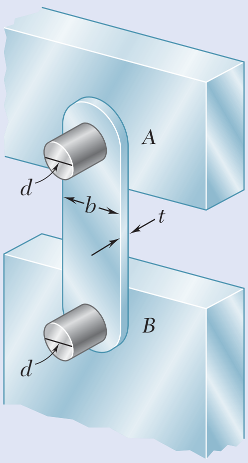


Fig. P1.23

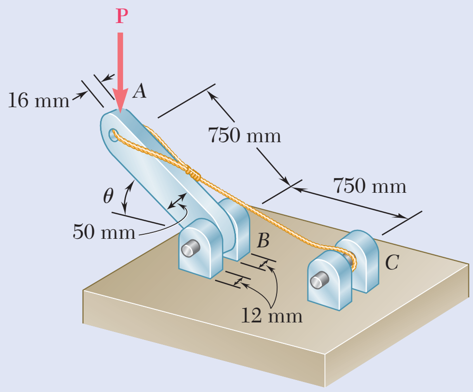


Fig. P1.24 and P1.25

1.24 Determine the largest load **P** that can be applied at A when $\theta = 60°$, knowing that the average shearing stress in the 10-mm-diameter pin at B must not exceed 120 MPa and that the average bearing stress in member AB and in the bracket at B must not exceed 90 MPa.

1.25 Knowing that $\theta = 40°$ and $P = 9$ kN, determine (*a*) the smallest allowable diameter of the pin at B if the average shearing stress in the pin is not to exceed 120 MPa, (*b*) the corresponding average bearing stress in member AB at B, (*c*) the corresponding average bearing stress in each of the support brackets at B.

1.26 The hydraulic cylinder CF, which partially controls the position of rod DE, has been locked in the position shown. Member BD is 15 mm thick and is connected at C to the vertical rod by a 9-mm-diameter bolt. Knowing that $P = 2$ kN and $\theta = 75°$, determine (*a*) the average shearing stress in the bolt, (*b*) the bearing stress at C in member BD.

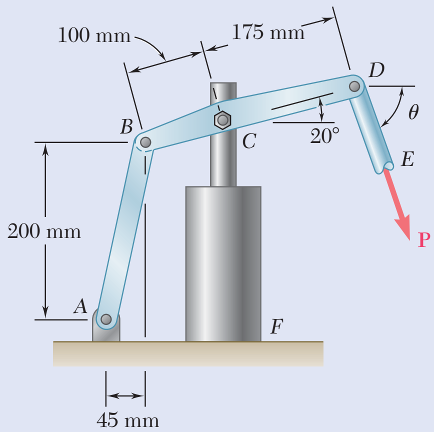


Fig. *P1.26*

1.27 For the assembly and loading of Prob. 1.7, determine (*a*) the average shearing stress in the pin at B, (*b*) the average bearing stress at B in member BD, (*c*) the average bearing stress at B in member ABC, knowing that this member has a 10×50-mm uniform rectangular cross section.

1.28 Two identical linkage-and-hydraulic-cylinder systems control the position of the forks of a fork-lift truck. The load supported by the one system shown is 1500 lb. Knowing that the thickness of member BD is $\frac{5}{8}$ in., determine (*a*) the average shearing stress in the $\frac{1}{2}$-in.-diameter pin at B, (*b*) the bearing stress at B in member BD.

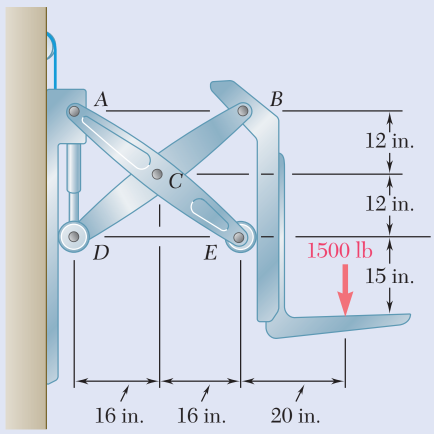


Fig. P1.28

1.3 STRESS ON AN OBLIQUE PLANE UNDER AXIAL LOADING

Previously, axial forces exerted on a two-force member (Fig. 1.26*a*) caused normal stresses in that member (Fig. 1.26*b*), while transverse forces exerted on bolts and pins (Fig. 1.27*a*) caused shearing stresses in those connections (Fig. 1.27*b*). Such a relation was observed between axial forces and normal stresses and transverse forces and shearing stresses, because stresses were being determined only on planes perpendicular to the axis of the member or connection. In this section, axial forces cause both normal and shearing stresses on planes that are not perpendicular to the axis of the member. Similarly, transverse forces exerted on a bolt or a pin cause both normal and shearing stresses on planes that are not perpendicular to the axis of the bolt or pin.

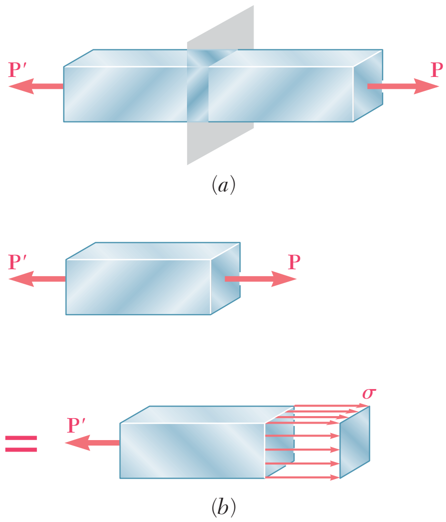


Fig. 1.26 Axial forces on a two-force member. (*a*) Section plane perpendicular to member away from load application. (*b*) Equivalent force diagram models of resultant force acting at centroid and uniform normal stress.

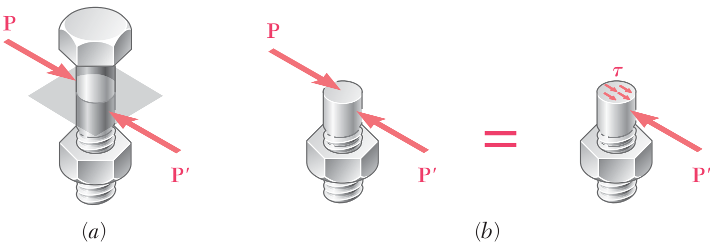


Fig. 1.27 (*a*) Diagram of a bolt from a single-shear joint with a section plane normal to the bolt. (*b*) Equivalent force diagram models of the resultant force acting at the section centroid and the uniform average shear stress.

Consider the two-force member of Fig. 1.26 that is subjected to axial forces **P** and **P**′. If we pass a section forming an angle θ with a normal plane (Fig. 1.28*a*) and draw the free-body diagram of the portion of member located to the left of that section (Fig. 1.28*b*), the equilibrium conditions of the free body show that the distributed forces acting on the section must be equivalent to the force **P.**

Resolving **P** into components **F** and **V**, respectively normal and tangential to the section (Fig. 1.28*c*),

$$F = P\cos\theta \qquad V = P\sin\theta \tag{1.12}$$

Force **F** represents the resultant of normal forces distributed over the section, and force **V** is the resultant of shearing forces (Fig. 1.28*d*). The average values of the corresponding normal and shearing stresses are obtained by dividing F and V by the area A_θ of the section:

$$\sigma = \frac{F}{A_\theta} \qquad \tau = \frac{V}{A_\theta} \tag{1.13}$$

Substituting for F and V from Eq. (1.12) into Eq. (1.13), and observing from Fig. 1.28*c* that $A_0 = A_\theta \cos\theta$ or $A_\theta = A_0/\cos\theta$, where A_0 is the area of a section perpendicular to the axis of the member, we obtain

$$\sigma = \frac{P\cos\theta}{A_0/\cos\theta} \qquad \tau = \frac{P\sin\theta}{A_0/\cos\theta}$$

or

$$\sigma = \frac{P}{A_0}\cos^2\theta \qquad \tau = \frac{P}{A_0}\sin\theta\cos\theta \tag{1.14}$$

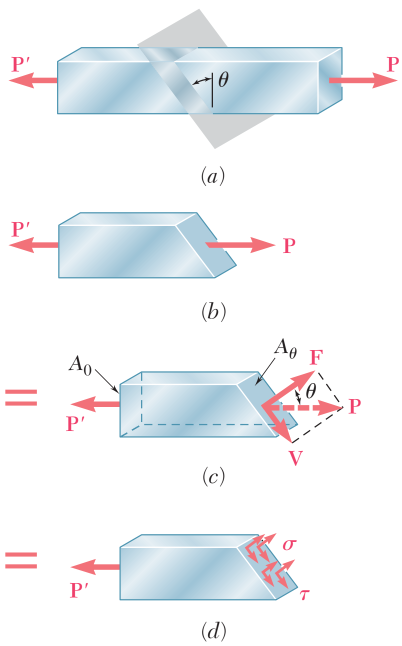


Fig. 1.28 Oblique section through a two-force member. (*a*) Section plane made at an angle θ to the member normal plane, (*b*) Free-body diagram of left section with internal resultant force **P**. (*c*) Free-body diagram of resultant force resolved into components **F** and **V** along the section plane's normal and tangential directions, respectively. (*d*) Free-body diagram with section forces **F** and **V** represented as normal stress, σ, and shearing stress, τ.

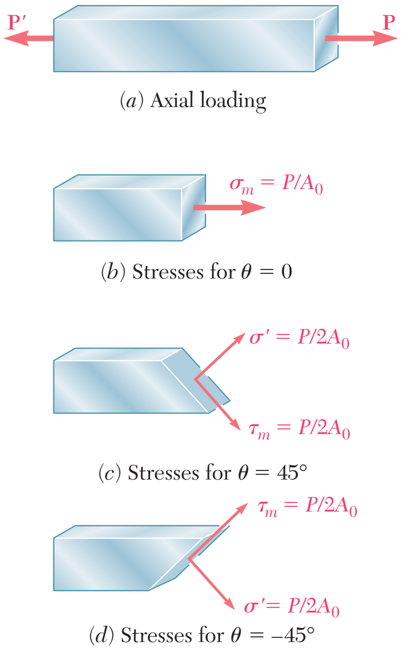


Fig. 1.29 Selected stress results for axial loading.

Note from the first of Eqs. (1.14) that the normal stress σ is maximum when $\theta = 0$ (i.e., the plane of the section is perpendicular to the axis of the member). It approaches zero as θ approaches 90°. We check that the value of σ when $\theta = 0$ is

$$\sigma_m = \frac{P}{A_0} \tag{1.15}$$

The second of Eqs. (1.14) shows that the shearing stress τ is zero for $\theta = 0$ and $\theta = 90°$. For $\theta = 45°$, it reaches its maximum value

$$\tau_m = \frac{P}{A_0} \sin 45° \cos 45° = \frac{P}{2A_0} \tag{1.16}$$

The first of Eqs. (1.14) indicates that, when $\theta = 45°$, the normal stress σ' is also equal to $P/2A_0$:

$$\sigma' = \frac{P}{A_0} \cos^2 45° = \frac{P}{2A_0} \tag{1.17}$$

The results obtained in Eqs. (1.15), (1.16), and (1.17) are shown graphically in Fig. 1.29. The same loading may produce either a normal stress $\sigma_m = P/A_0$ and no shearing stress (Fig. 1.29*b*) or a normal and a shearing stress of the same magnitude $\sigma' = \tau_m = P/2A_0$ (Fig. 1.29*c* and *d*), depending upon the orientation of the section.

1.4 STRESS UNDER GENERAL LOADING CONDITIONS; COMPONENTS OF STRESS

The examples of the previous sections were limited to members under axial loading and connections under transverse loading. Most structural members and machine components are under more involved loading conditions.

Consider a body subjected to several loads $\mathbf{P}_1$, $\mathbf{P}_2$, etc. (Fig. 1.30). To understand the stress condition created by these loads at some point Q within the body, we shall first pass a section through Q, using a plane parallel to the yz plane. The portion of the body to the left of the section is subjected to some of the original loads, and to normal and shearing forces distributed over the section. We shall denote by $\Delta\mathbf{F}^x$ and $\Delta\mathbf{V}^x$, respectively, the normal and the shearing forces acting on a small area ΔA surrounding point Q (Fig. 1.31*a*). Note that the superscript x is used to indicate that the forces $\Delta\mathbf{F}^x$ and $\Delta\mathbf{V}^x$ act on a surface perpendicular to the x axis. While the normal force $\Delta\mathbf{F}^x$ has a well-defined direction, the shearing force $\Delta\mathbf{V}^x$ may have any direction in the plane of the section. We therefore resolve $\Delta\mathbf{V}^x$ into two component forces, $\Delta\mathbf{V}^x_y$ and $\Delta\mathbf{V}^x_z$, in directions parallel to the y and z axes, respectively (Fig. 1.31*b*). Dividing the magnitude of each force by the area ΔA and letting ΔA approach zero, we define the three stress components shown in Fig. 1.32:

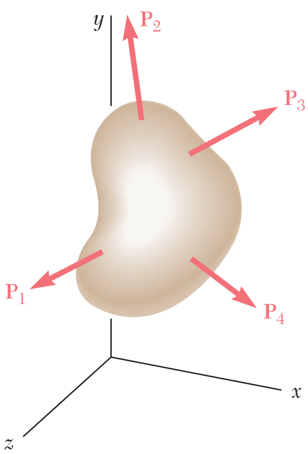


Fig. 1.30 Multiple loads on a general body.

$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F^x}{\Delta A}$$
$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta V^x_y}{\Delta A} \qquad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta V^x_z}{\Delta A} \tag{1.18}$$

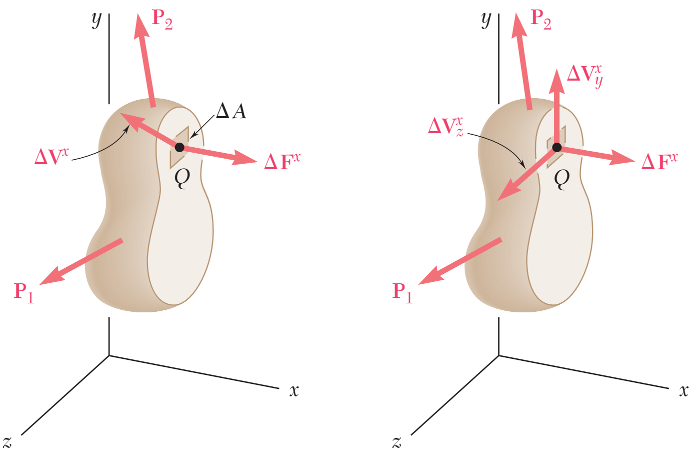


Fig. 1.31 (*a*) Resultant shear and normal forces, $\Delta\mathbf{V}^x$ and $\Delta\mathbf{F}^x$, acting on small area ΔA at point Q. (b) Forces on ΔA resolved into forces in coordinate directions.

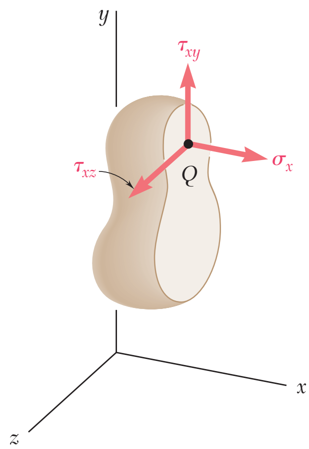


Fig. 1.32 Stress components at point Q on the body to the left of the plane.

Note that the first subscript in σ_x, τ_{xy}, and τ_{xz} is used to indicate that the stresses are exerted *on a surface perpendicular to the x axis.* The second subscript in τ_{xy} and τ_{xz} identifies *the direction of the component.* The normal stress σ_x is positive if the corresponding arrow points in the positive x direction (i.e., if the body is in tension) and negative otherwise. Similarly, the shearing stress components τ_{xy} and τ_{xz} are positive if the corresponding arrows point, respectively, in the positive y and z directions.

This analysis also may be carried out by considering the portion of body located to the right of the vertical plane through Q (Fig. 1.33). The same magnitudes, but opposite directions, are obtained for the normal and shearing forces $\Delta\mathbf{F}^x$, $\Delta\mathbf{V}_y^x$, and $\Delta\mathbf{V}_z^x$. Therefore, the same values are obtained for the corresponding stress components. However as the section in Fig. 1.33 now faces the *negative x axis,* a positive sign for σ_x indicates that the corresponding arrow points *in the negative x direction.* Similarly, positive signs for τ_{xy} and τ_{xz} indicate that the corresponding arrows point in the negative y and z directions, as shown in Fig. 1.33.

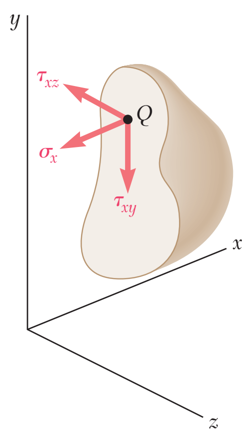


Fig. 1.33 Stress components at point Q on the body to the right of the plane.

Passing a section through Q parallel to the zx plane, we define the stress components, σ_y, τ_{yz}, and τ_{yx}. Then, a section through Q parallel to the xy plane yields the components σ_z, τ_{zx}, and τ_{zy}.

To visualize the stress condition at point Q, consider a small cube of side a centered at Q and the stresses exerted on each of the six faces of the cube (Fig. 1.34). The stress components shown are σ_x, σ_y, and σ_z, which represent the normal stress on faces respectively perpendicular to the x, y, and z axes, and the six shearing stress components τ_{xy}, τ_{xz}, etc. Recall that τ_{xy} represents the y component of the shearing stress exerted on the face perpendicular to the x axis, while τ_{yx} represents the x component of the shearing stress exerted on the face perpendicular to the y axis. Note that only three faces of the cube are actually visible in Fig. 1.34 and that equal and opposite stress components act on the hidden faces. While the stresses acting on the faces of the cube differ slightly from the stresses at Q, the error involved is small and vanishes as side a of the cube approaches zero.

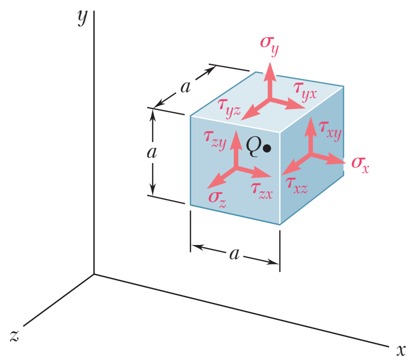


Fig. 1.34 Positive stress components at point Q.

Shearing stress components. Consider the free-body diagram of the small cube centered at point Q (Fig. 1.35). The normal and shearing forces acting on the various faces of the cube are obtained by multiplying the corresponding stress components by the area ΔA of each face. First write the following three equilibrium equations

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0 \tag{1.19}$$

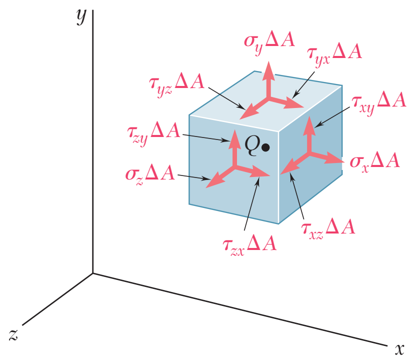


Fig. 1.35 Positive resultant forces on a small element at point Q resulting from a state of general stress.

Since forces equal and opposite to the forces actually shown in Fig. 1.35 are acting on the hidden faces of the cube, Eqs. (1.19) are satisfied. Considering the moments of the forces about axes x', y', and z' drawn from Q in directions respectively parallel to the x, y, and z axes, the three additional equations are

$$\Sigma M_{x'} = 0 \qquad \Sigma M_{y'} = 0 \qquad \Sigma M_{z'} = 0 \tag{1.20}$$

Using a projection on the $x'y'$ plane (Fig. 1.36), note that the only forces with moments about the z axis different from zero are the shearing forces. These forces form two couples: a counterclockwise (positive) moment $(\tau_{xy}\,\Delta A)a$ and a clockwise (negative) moment $-(\tau_{yx}\,\Delta A)a$. The last of the three Eqs. (1.20) yields

$$+\circlearrowleft \Sigma M_z = 0: \qquad (\tau_{xy}\,\Delta A)a - (\tau_{yx}\,\Delta A)a = 0$$

from which

$$\tau_{xy} = \tau_{yx} \tag{1.21}$$

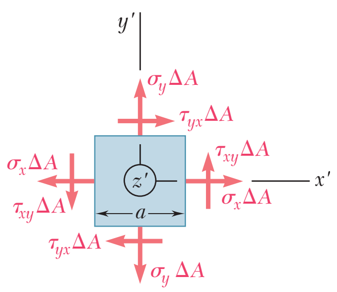


Fig. 1.36 Free-body diagram of small element at Q viewed on projected plane perpendicular to z'-axis. Resultant forces on positive and negative z' faces (not shown) act through the z'-axis, thus do not contribute to the moment about that axis.